Effects of Prandtl's Secondary Flows of the Second Kind on sCO_2 Heat Transfer

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MODEL VALIDATION

Boussinesq Approach (Isotropic)

$$\tau_{ij} = -\rho\overline{u_i'' u_j''} \rightarrow 2\mu_t S_{ij} - \frac{2}{3}\rho k \delta_{ij}$$

$$\tau_{ij} = -\rho\overline{u_i'' u_j''} \rightarrow \frac{D(\rho\overline{u_i'' u_j''})}{Dt} = -D_{T,ij} + D_{L,ij} - P_{ij} + \varphi_{ij} - \varepsilon_{ij}$$

Reynolds Stress Tensor

RSM Approach (Anisotropic)

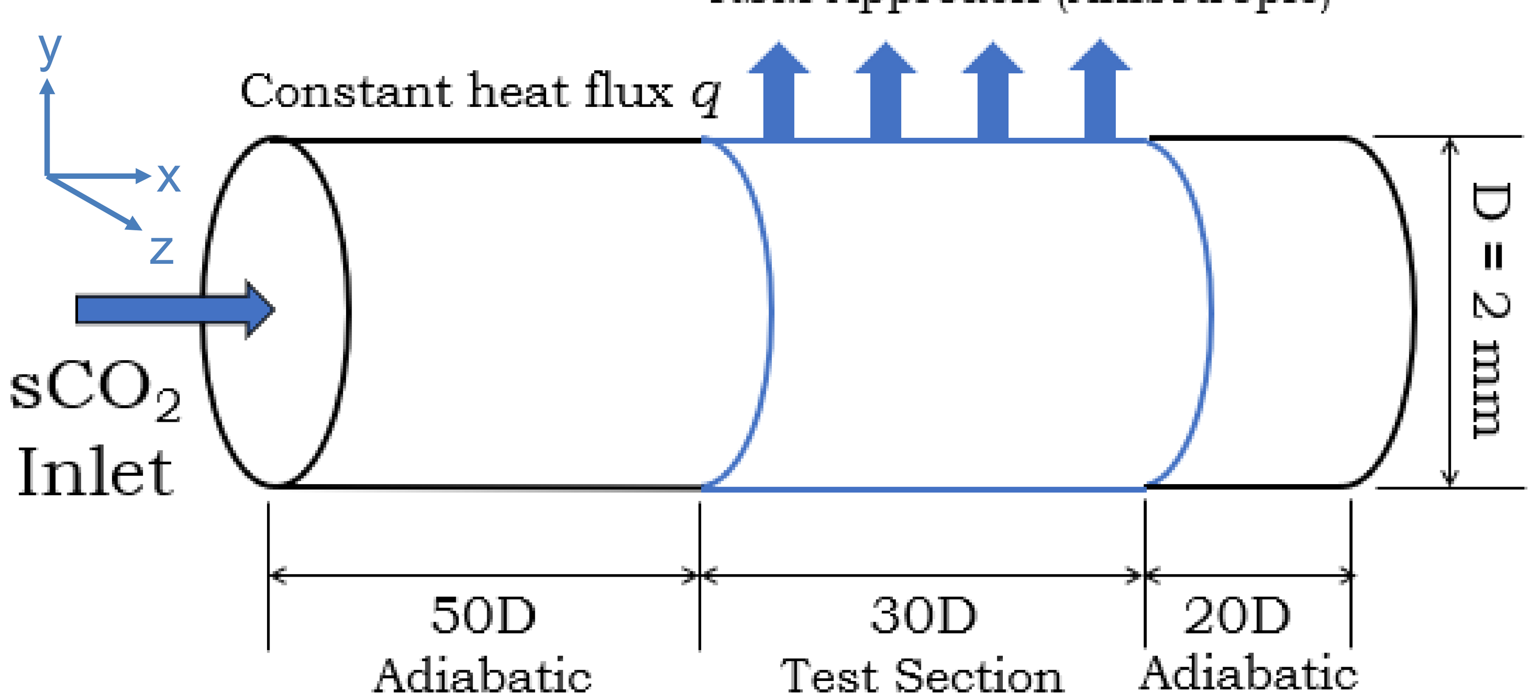

Figure 3: Circular pipe geometry for model validation

- Used anisotropic RANS model called Reynolds Stress Model (RSM) that solves transport equations for each component of the Reynolds stress tensor
- Validated RSM against Direct Numerical Simulation (DNS) from literature with sCO_2 in a circular duct

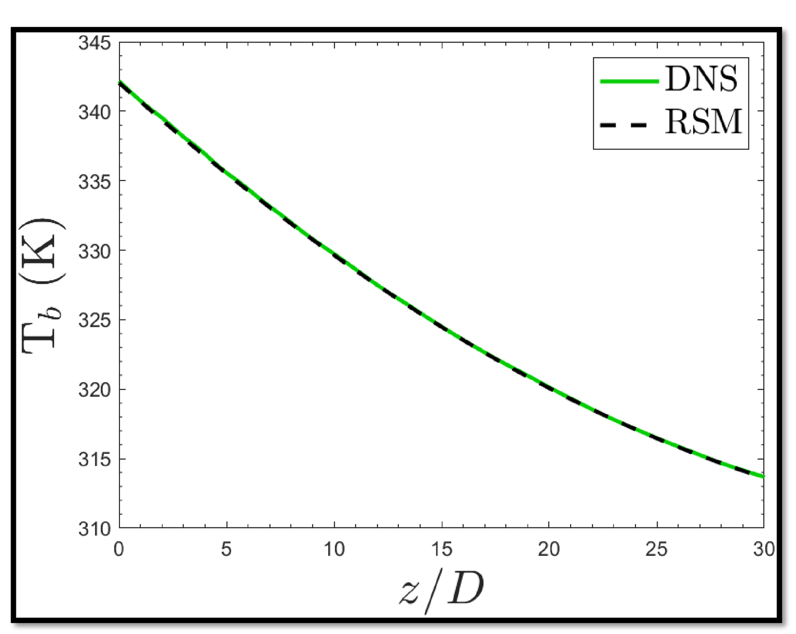

Figure 4: Bulk temperature model data

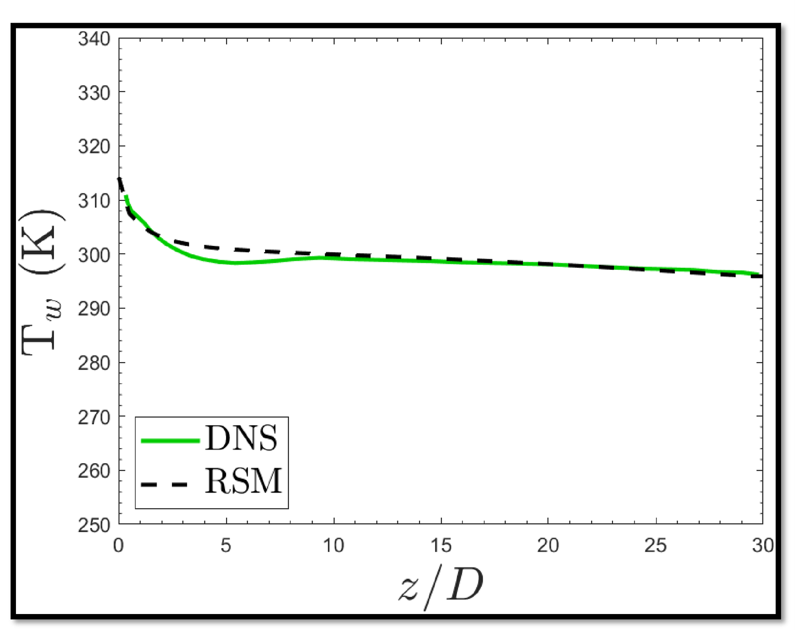

Figure 5: Wall temperature model data

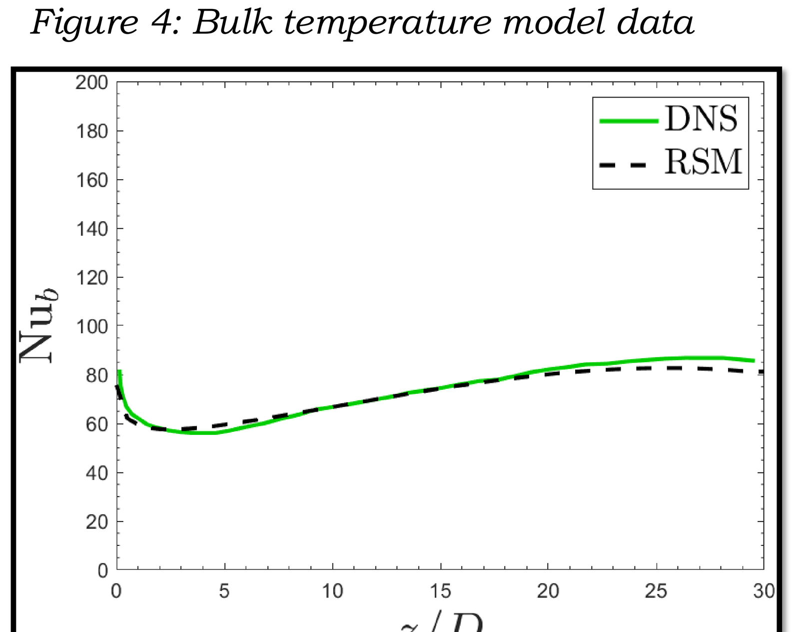

Figure 6: Bulk Nusselt number data

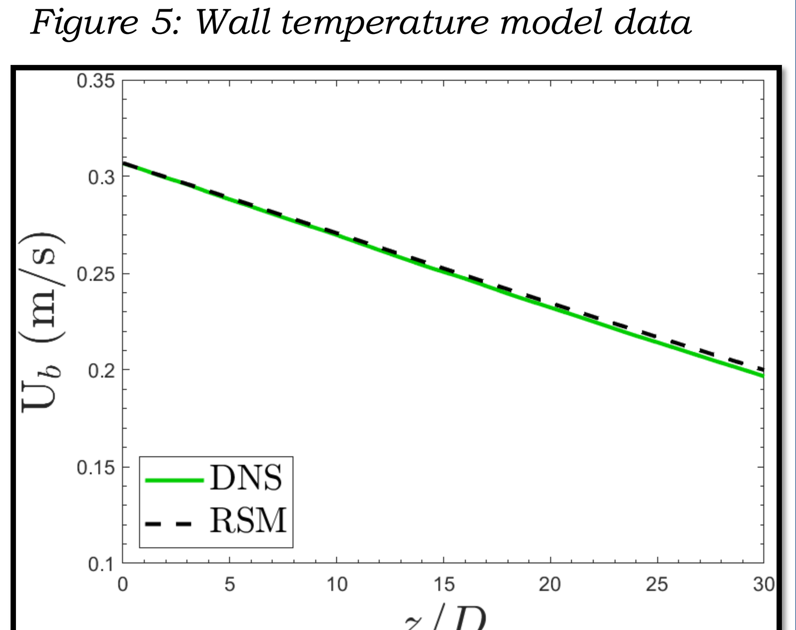

Figure 7: Bulk velocity model data

INTRODUCTION

Supercritical Carbon Dioxide (sCO_2) is a promising heat transfer fluid due to its favorable yet complex thermodynamic properties near the critical point. Due to its complexity and usability in turbulent flows, numerical modeling with turbulence must be used as a predictive tool for sCO_2 development in engineering applications.

Standard Reynolds-Averaged Navier Stokes (RANS) turbulence models assume isotropic Reynolds stresses, which fail to capture the Prandtl's Secondary Flows of the Second Kind present within internal flows in corners. This study explores an anisotropic turbulence model to quantify the heat transfer effect considering these secondary flows compared to the heat transfer under the isotropic assumption.

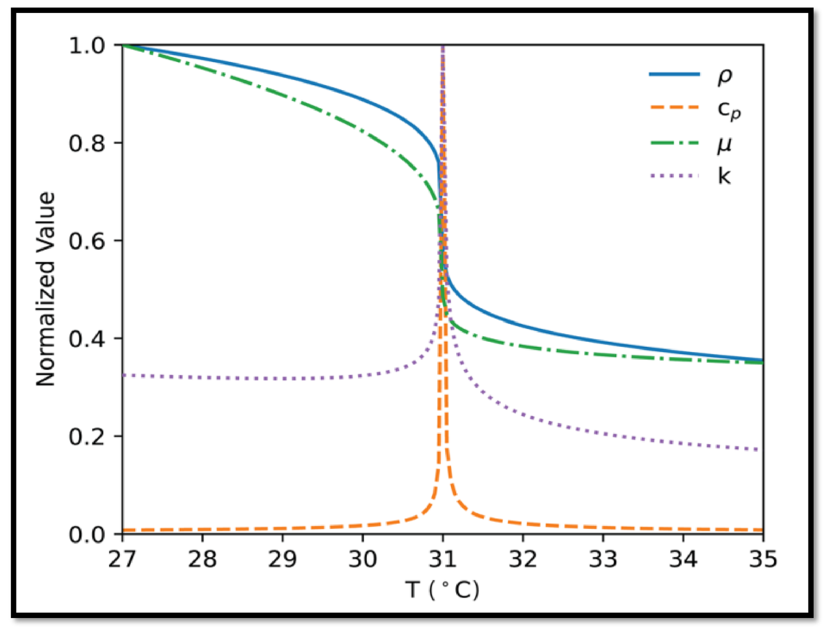

Figure 1: Thermophysical properties of sCO2 near the critical point from Lopes et al. (10.1115/1.4055345)

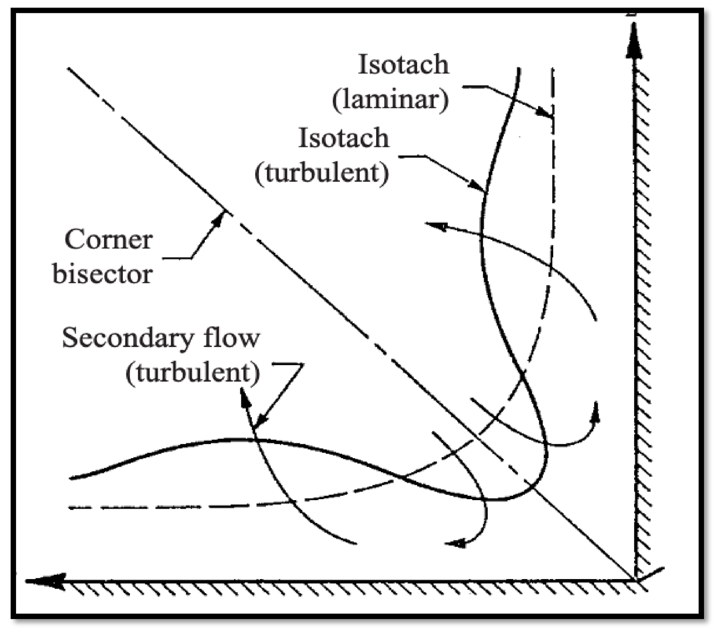

Figure 2: Schematic of warped isotachs from Moinuddin et al. (10.1017/s0022112004008742)

CORNER EFFECTS

- Secondary flow works to redistribute momentum and energy across the duct cross-section outside the viscous sublayer, impacting heat transfer
- Measured secondary velocity magnitude was ~1-2% of mean streamwise flow velocity
- Circulation causes two pair of counter-rotating vortices along the corner bisector of the square duct

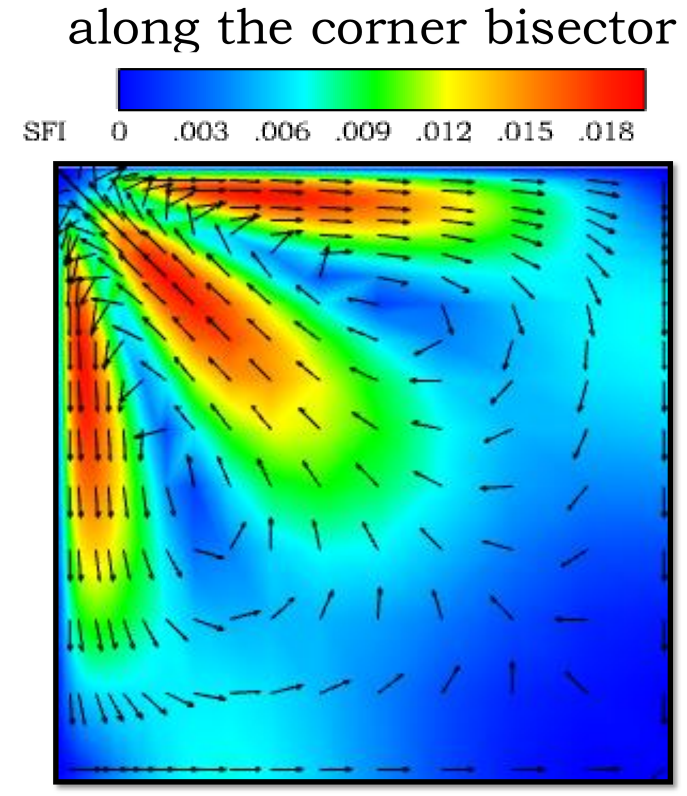

Figure 13: Magnitude of the secondary flow velocity in the corner of the duct

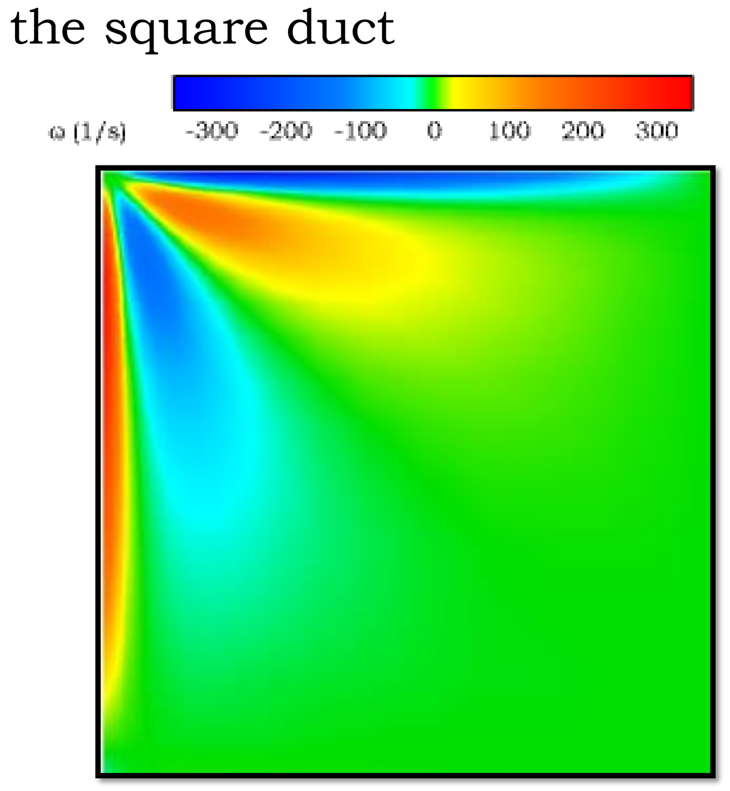

Figure 14: Magnitude of the streamwise vorticity in the corner of the duct

HEAT TRANSFER DATA

Heat transfer data from the RSM is compared to the isotropic k-ω SST RANS model both with and without a corner correction setting in a square duct with similar turbulent Prandtl number fine-tuning.

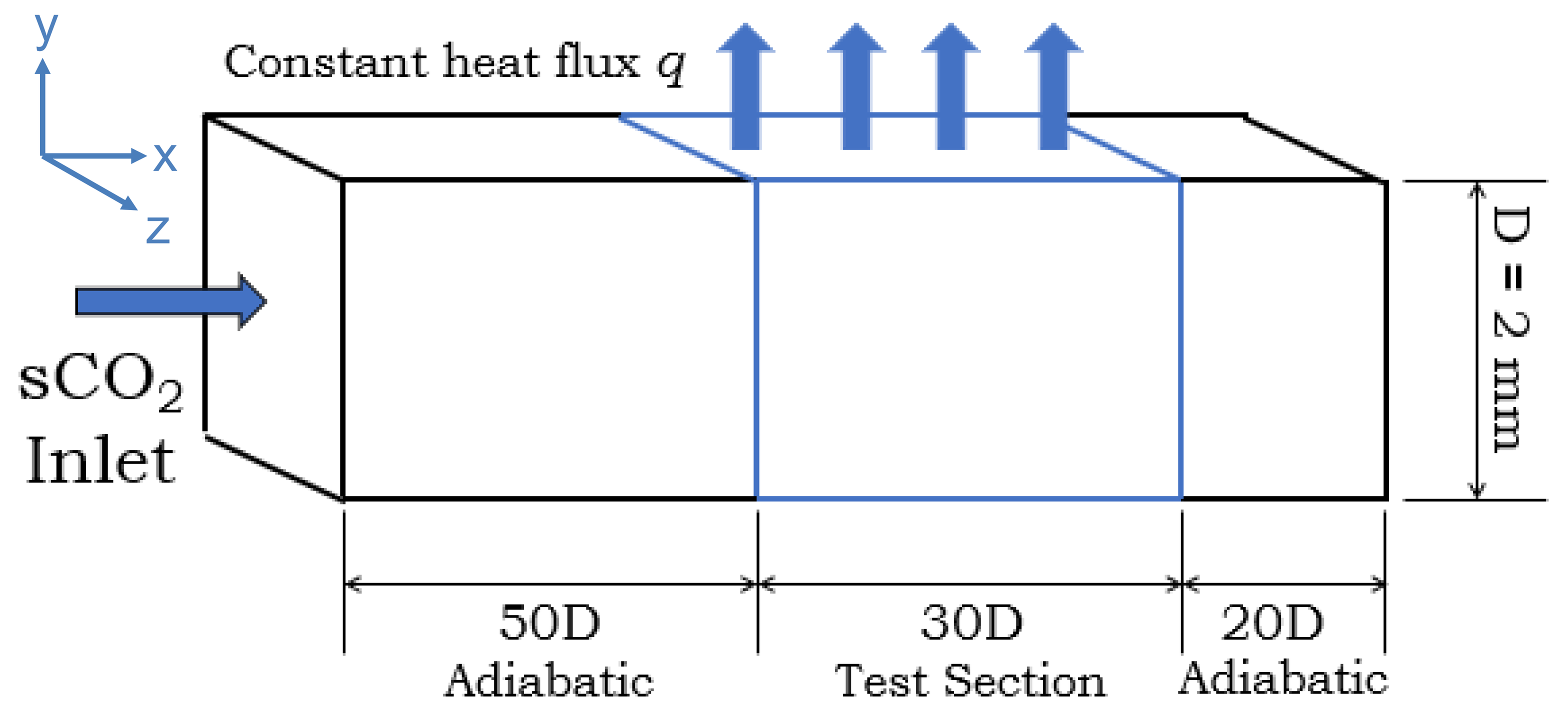

Figure 8: Square pipe geometry for RSM & SST heat transfer

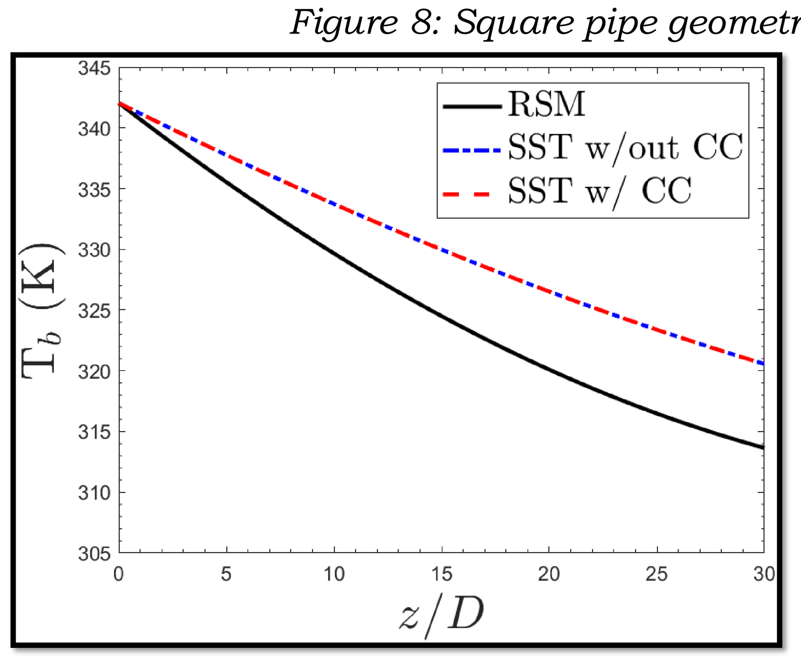

Figure 9: Bulk temperature model data

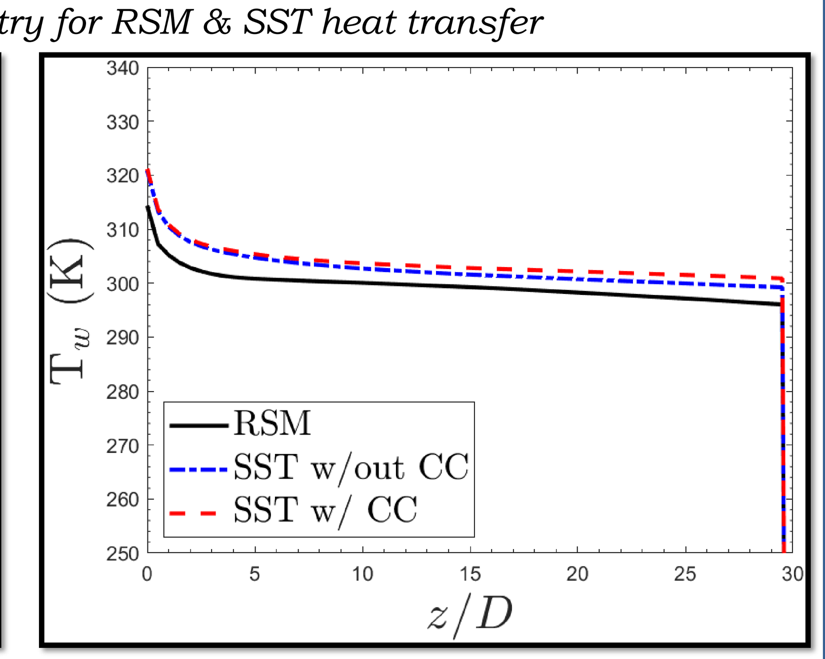

Figure 10: Wall temperature model data

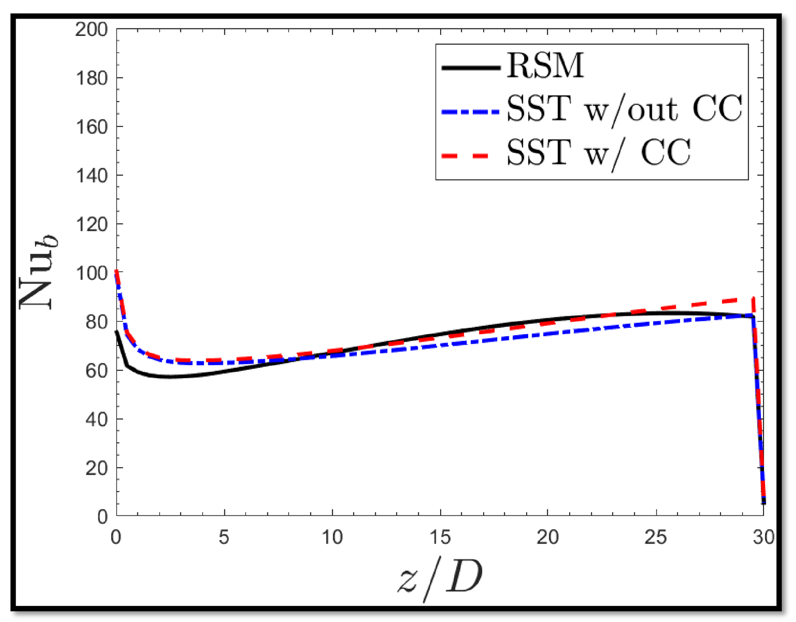

Figure 11: Bulk Nusselt number data

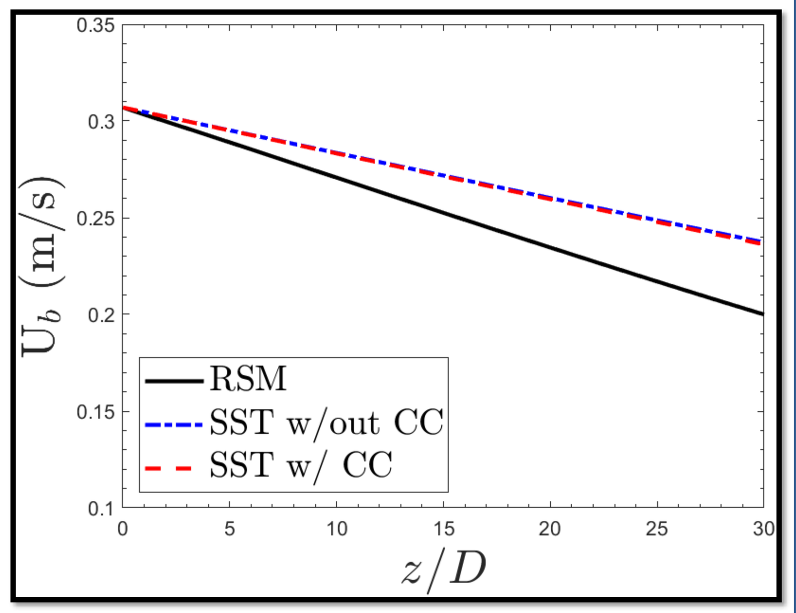

Figure 12: Bulk velocity model data

CONCLUSION

Within the current validation capabilities of existing sCO_2 literature, it can be concluded that the RSM is a direct improvement over isotropic models in capturing heat transfer effects due to anisotropy. The RSM shows high promise in producing quality results with sCO_2 over SST by better replicating DNS data in a circular duct with less parameter fine-tuning and recording up to a 7 Kelvin difference in bulk temperature data against SST in a square duct.

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