# An Improved Off-Design Prediction Model Based on Similarity Analysis for Turbomachinery Under S-$CO_2$ Condition

Presenter : Yongju Jeong

Yongju Jeong, Seongmin Son, Seong Kuk Cho, Jeong Ik Lee

*Dept. of Nuclear & Quantum Engineering, KAIST*

# Contents

# Background – S-$CO_2$ Power cycle

## Steam Rankine Cycle

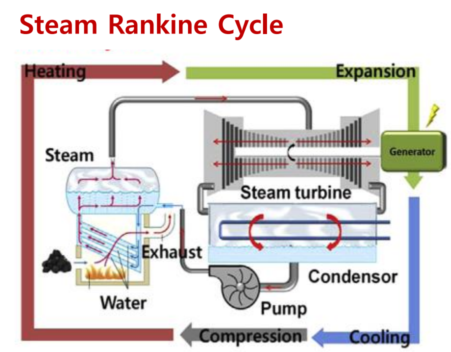
- Working fluid : Water
- Negligible pumping work
- Very large expansion ratio
- Large turbine and system size
- Phase change over the cycle
- Relatively low heat source temp

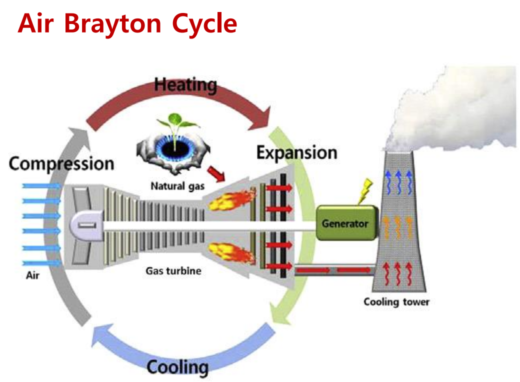
## Air Brayton Cycle

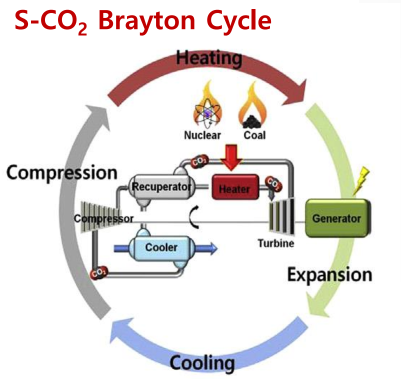
- Working fluid : Air
- Comp. vs Turb. work ratio is high
- Compact turbomachinery
- No phase change over the cycle
- High heat source temperature
- Not only for power plant but also for propulsion application

## S-$CO_2$ Brayton Cycle

- Working fluid : $CO_2$
- Reduced compression work
- Compact turbomachinery
- Low expansion ratio with recuperation
- No phase change over the cycle
- Wide range of heat source temp
- Relatively high heat sink temp







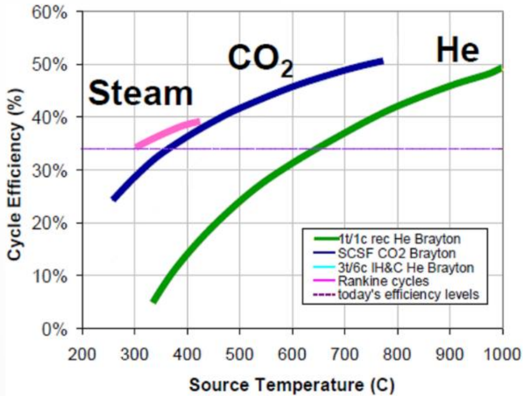
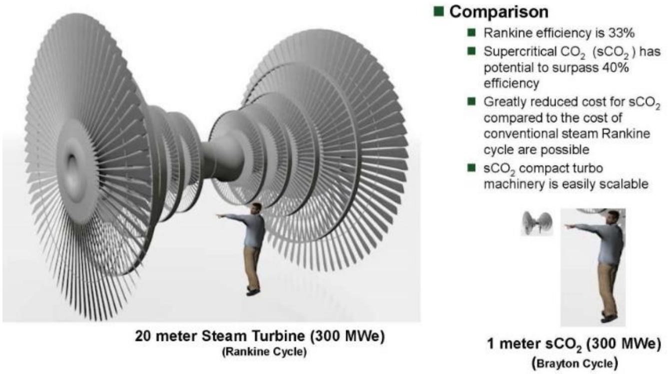


Fig. Cycle thermal efficiency with varying source temperature



- **Comparison**
  - Rankine efficiency is 33%
  - Supercritical $CO_2$ (s$CO_2$) has potential to surpass 40% efficiency
  - Greatly reduced cost for s$CO_2$ compared to the cost of conventional steam Rankine cycle are possible
  - s$CO_2$ compact turbo machinery is easily scalable

**20 meter Steam Turbine (300 MWe)**
**(Rankine Cycle)**

**1 meter s$CO_2$ (300 MWe)**
**(Brayton Cycle)**

Fig. Size comparison between steam and S-$CO_2$ cycle

# Background – S-$CO_2$ Power cycle

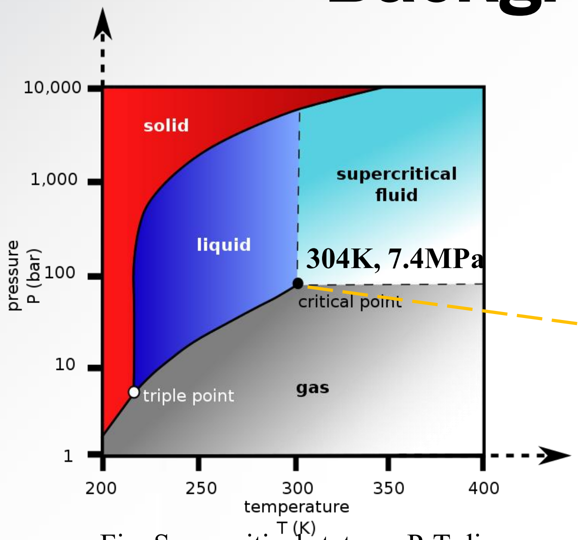
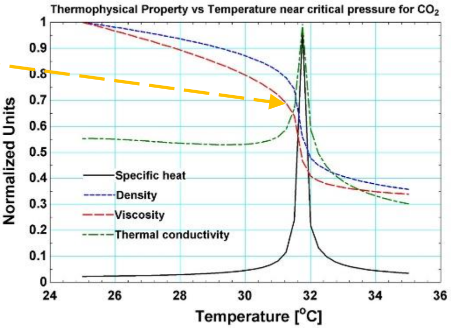


Fig. Supercritical state on P-T diagram



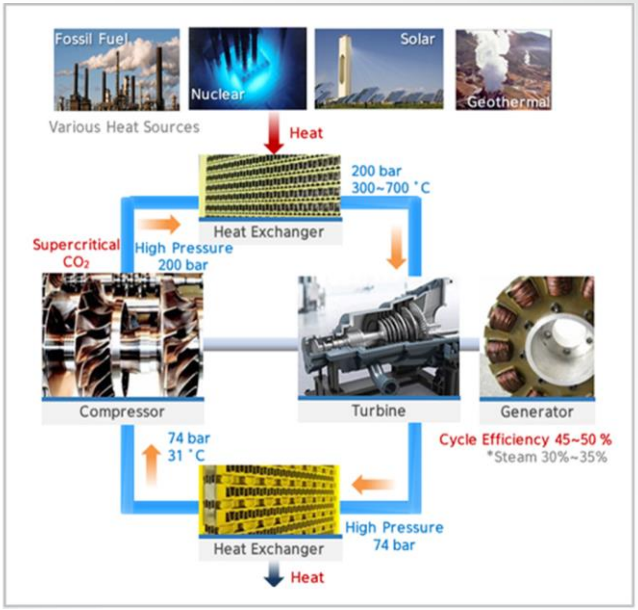


Fig. Applications of S-$CO_2$ power cycle

- S-$CO_2$ power cycle is a variation of Brayton cycle
- Operates above the critical point
- Why supercritical '$CO_2$' cycle? ➔ Relatively moderate critical point ($T_c$, $P_c$)
- Drastic fluid property changes near the critical point
- Favorable fluid property in the critical region
  - Liquid like high density but low viscosity, large specific heat($C_p$)
  - Reduced compressibility near the critical point ➔ reduced compression work
- Applicability for various heat source over wide range of source temperatures
  ; nuclear reactor, concentrated solar power (CSP), clean fossil fuel, waste heat recovery
- High efficiency, compact and simple layout

# Background – Off-design Performance Prediction

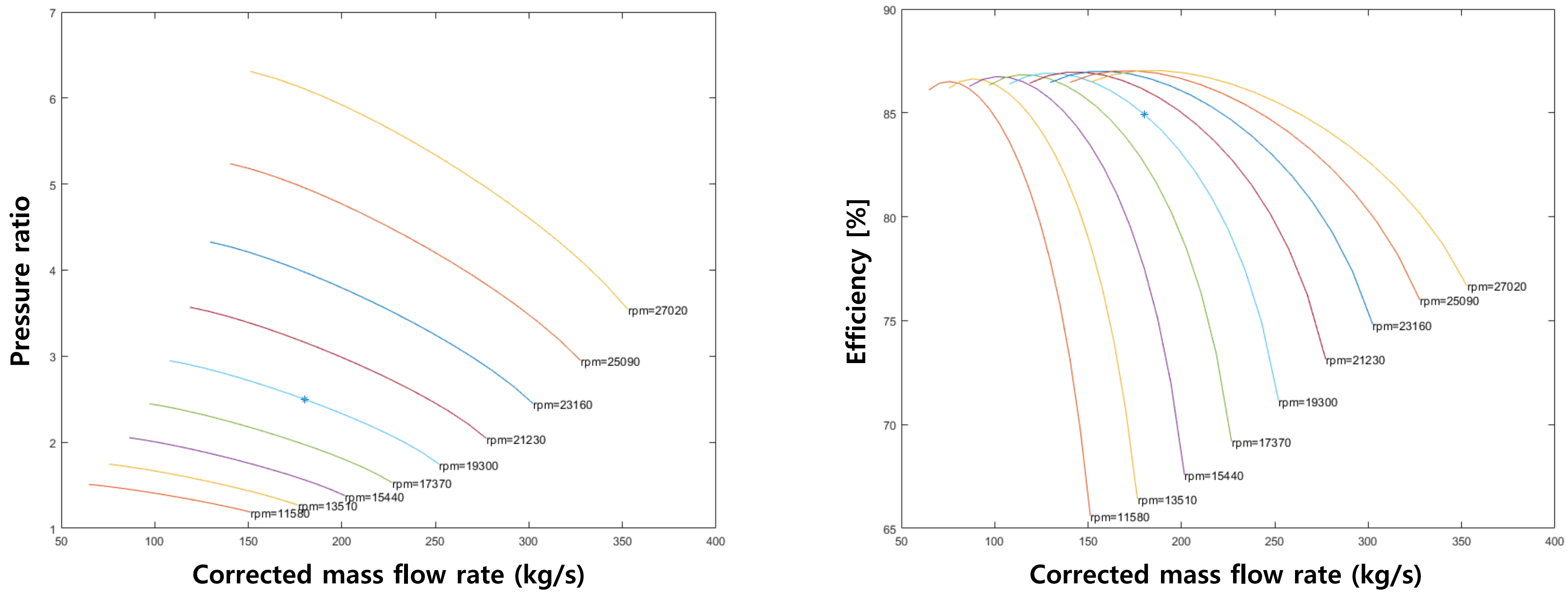


Fig. Compressor performance map example for a fixed inlet condition

- Compressor off-design operation → $[\dot{m}] * [rpm] * [T_{in}] * [P_{in}]$; $n^4$ calculation
- Impractical to calculate all cases according to the off-design operation variables
- Thus, the similarity correction method has been used for a conventional air compressor

$[\dot{m}] * [rpm] * \boxed{[T_{in}] * [P_{in}]}$ → simplified to 'corrected mass flow rate' & 'corrected rpm'

**Effect of $T_{in}$, $P_{in}$ variation into mass flow rate, rpm**

- Compressor performance map → compressor performance $[PR]$ or $[Eff]$ vs. $[\dot{m}] * [rpm]$

# Literature Review

## Dimensional analysis and derivation

### ❖ Ideal gas model (IG)

- Thinking compressor as a function, fn $(D, N, m, P_{in}, T_{in}, R, \gamma)=P_{out}, \Delta H, T_{out}$
- Applying dimensional analysis (Buckingham Pi theorem) with the ideal gas assumption,

$$\mathrm{fn}\left(\boxed{\frac{m\sqrt{\gamma RT_{in}}}{D^2\gamma P_{in}}, \frac{ND}{\sqrt{\gamma RT_{in}}}}\right) = \frac{P_{out}}{P_{in}}, \frac{\Delta H}{\sqrt{\gamma RT_{in}}}, \frac{T_{out}}{T_{in}} \qquad \mu = \frac{T_{out\ isen} - Tin}{T_{out} - Tin} = \frac{(P_{out}/Pin)^{(\gamma-1)/\gamma} - 1}{(Tout/Tin) - 1}$$

**Dimensionless mass flow rate and rpm**

- As far as same compressor is used, D can be neglected

Flow parameter : $\Pi_1 = \frac{m\sqrt{\gamma RT_{in}}}{\gamma P_{in}}$

Head parameter : $\Pi_3 = \frac{m\sqrt{\gamma RT_{in}}}{\gamma P_{in}}$

Efficiency parameter : $\Pi_5 = \eta$

Speed parameter : $\Pi_2 = \frac{N}{\sqrt{\gamma RT_{in}}}$

Pressure parameter : $\Pi_4 = PR$

Same $\Pi_1$, $\Pi_2$ → should have same compressor performance $(\Pi_3, \Pi_4, \Pi_5)$

- $\left(\frac{m\sqrt{\gamma RT_{in}}}{\gamma P_{in}}\right)_{ref} = \left(\frac{m\sqrt{\gamma RT_{in}}}{\gamma P_{in}}\right)_{off}$

→ $m_{cor} = m_{ref} = \left(\frac{m\sqrt{\gamma RT_{in}}}{\gamma P_{in}}\right)_{off} / \left(\frac{\sqrt{\gamma RT_{in}}}{\gamma P_{in}}\right)_{ref}$ ; corrected mass flow rate

- $\left(\frac{N}{\sqrt{\gamma RT_{in}}}\right)_{ref} = \left(\frac{N}{\sqrt{\gamma RT_{in}}}\right)_{off}$

→ $N_{cor} = N_{ref} = \left(\frac{N}{\sqrt{\gamma RT_{in}}}\right)_{off} / \left(\frac{N}{\sqrt{\gamma RT_{in}}}\right)_{ref}$ ; corrected rpm

# Literature Review

## Description of previously developed models

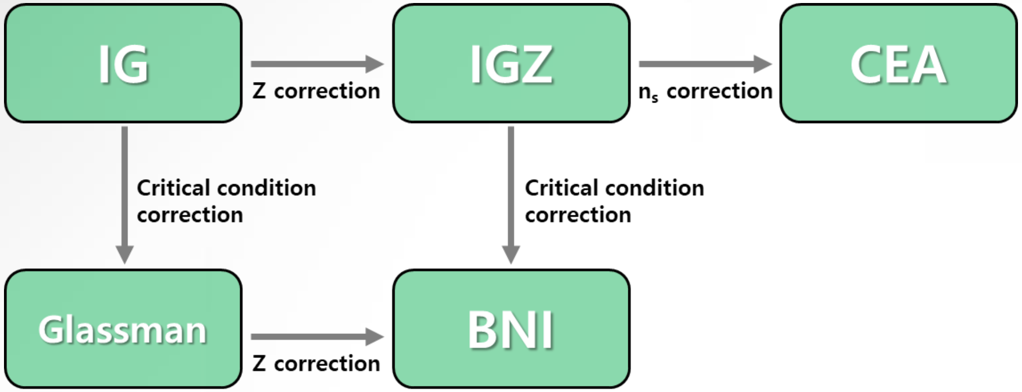

IG
Z correction
IGZ
$n_s$ correction
CEA
Critical condition correction
Critical condition correction
Glassman
Z correction
BNI

Fig. Summary of model development

| | Flow parameter | Speed parameter | Head parameter | Pressure parameter | Efficiency Parameter |
|---|---|---|---|---|---|
| **IG** | $\frac{\dot{m}\sqrt{\gamma RT}}{\gamma P}$ | $\frac{N}{\sqrt{\gamma RT}}$ | $\frac{\Delta H}{\gamma RT}$ | PR | η |
| **IGZ** | $\frac{\dot{m}\sqrt{\gamma ZRT}}{\gamma P}$ | $\frac{N}{\sqrt{\gamma ZRT}}$ | $\frac{\Delta H}{\gamma ZRT}$ | | |
| **Glassman** | $\frac{\dot{m}\sqrt{\gamma RT_{cr}}}{\gamma P_{cr}}$ | $\frac{N}{\sqrt{\gamma RT_{cr}}}$ | $\frac{\Delta H}{\gamma RT_{cr}}$ | | |
| **BNI** | $\frac{\dot{m}\sqrt{\gamma ZRT_{cr}}}{\gamma P_{cr}}$ | $\frac{N}{\sqrt{\gamma ZRT_{cr}}}$ | $\frac{\Delta H}{\gamma ZRT_{cr}}$ | | |
| **CEA** | $\frac{\dot{m}\sqrt{n_s ZRT}}{n_s P}$ | $\frac{N}{\sqrt{n_s ZRT}}$ | $\frac{\Delta H}{n_s ZRT}$ | | |

Table. Summary of similitude parameters

- IG model assumes the ideal gas law
- However, real gas effect should be considered
- Compressibility factor was reflected in IGZ model
- Glassman and BNI models adopted analogy of critical condition from IG and IGZ models

$$\frac{P_{cr}}{P_t} = \left(\frac{2}{\gamma+1}\right)^{\gamma/(\gamma-1)} \qquad \frac{T_{cr}}{T_t} = \left(\frac{2}{\gamma+1}\right)$$

- CEA model replaced specific heat ratio with isentropic exponent, which is more generalized parameters for real gas

$$pv^{\gamma} = \text{const} \;\rightarrow\; pv^{n_s} = \text{const}$$

$$\beta_T = -\frac{1}{v}\left(\frac{\partial v}{\partial p}\right)_T = \frac{1}{p} - \frac{1}{Z}\left(\frac{\partial Z}{\partial p}\right)_T$$

$$n_s = -\frac{v}{p}\left(\frac{\partial p}{\partial v}\right)_s = \frac{\gamma}{\beta_T p}$$

# Analysis Tool – KAIST-TMD

- KAIST-TMD is 1D mean streamline code for design and performance evaluation
- Loss model set for S-$CO_2$ compressors was selected, based on air condition
- Definition based total to static conversion is applied,
  because of highly non-linear behavior near the critical point of S-$CO_2$
- The code was validated against experiments (SCIEL, SNL)

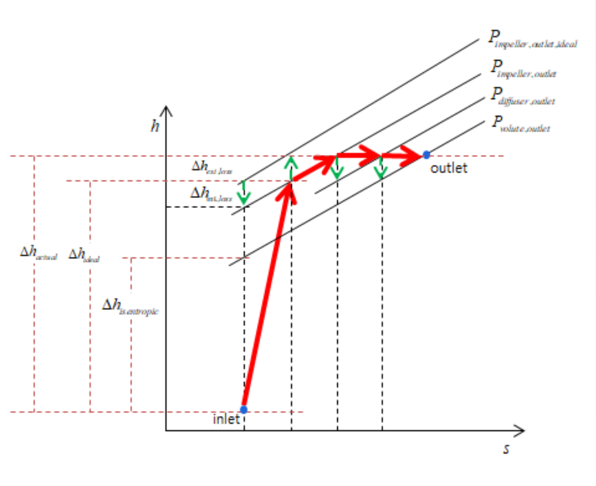
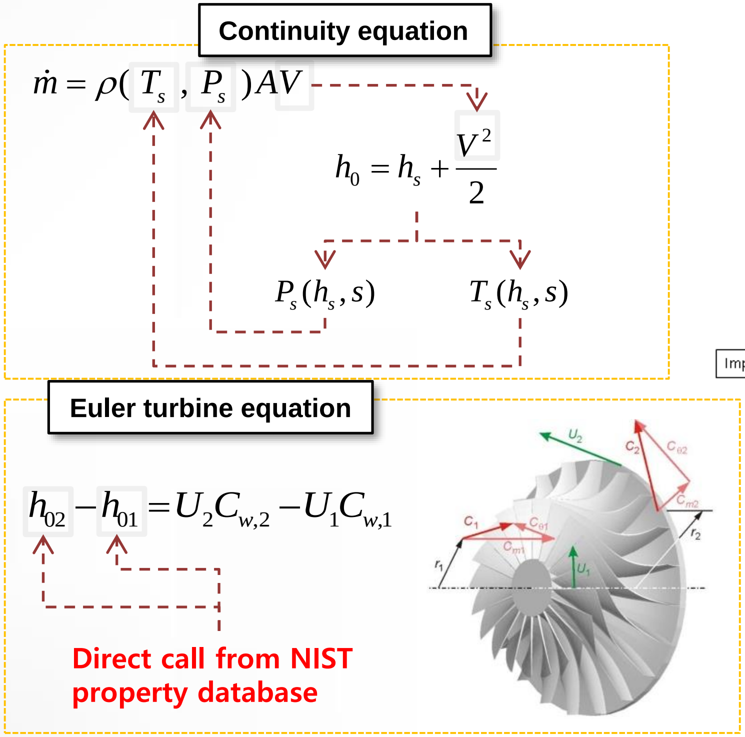


Fig. Mechanism of enthalpy loss model



**Continuity equation**

$$\dot{m} = \rho(T_s, P_s)AV$$

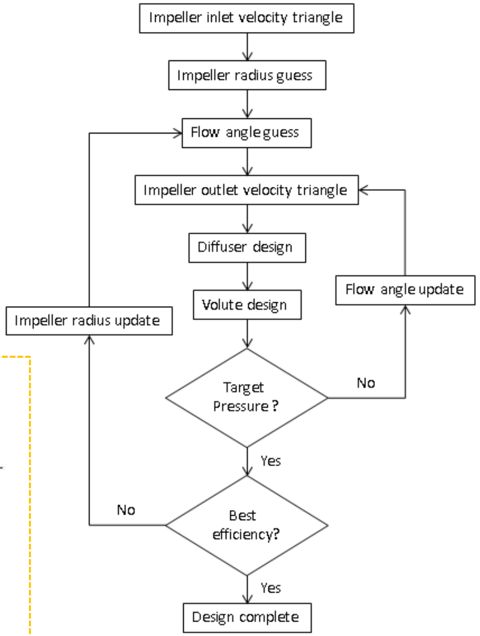
$$h_0 = h_s + \frac{V^2}{2}$$

$$P_s(h_s, s) \qquad T_s(h_s, s)$$

**Euler turbine equation**

$$h_{02} - h_{01} = U_2 C_{w,2} - U_1 C_{w,1}$$

Direct call from NIST property database



Impeller inlet velocity triangle
Impeller radius guess
Flow angle guess
Impeller outlet velocity triangle
Diffuser design
Volute design
Target Pressure ?
Yes
No
Flow angle update
Best efficiency?
Yes
No
Impeller radius update
Design complete

Fig. KAIST-TMD main algorithm

| Loss model | Oh set |
|---|---|
| **Incidence** | Conrad |
| **Blade loading** | Coppage |
| **Skin friction** | Jansen |
| **Mixing** | Johnston and Dean |
| **Clearance** | Jansen |
| **Disk friction** | Daily and Nece |
| **Recirculation** | Oh |
| **Leakage** | Aungier |

Fig. KAIST-TMD loss models

# Similitude Model Evaluation Method

## Error quantification and visualization

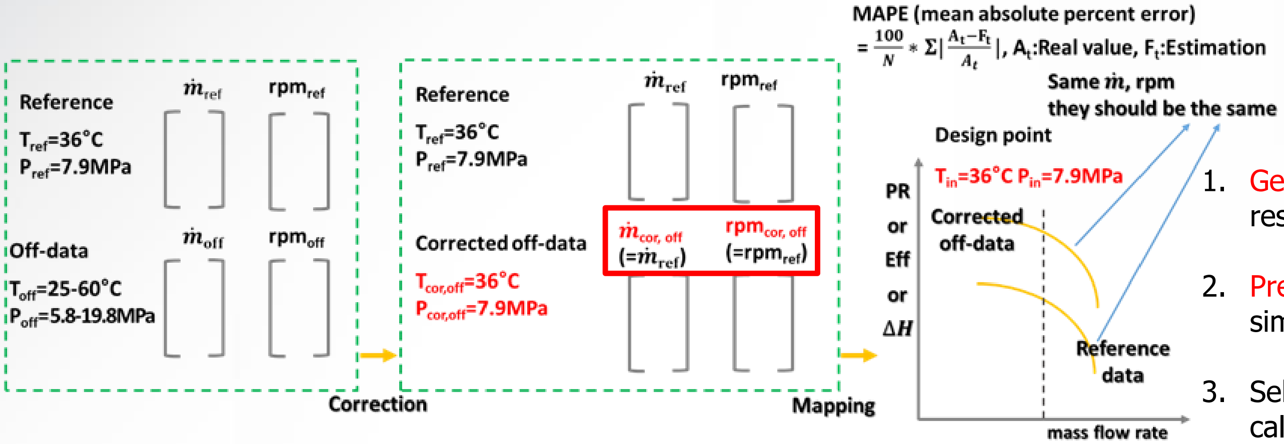


Fig. Flow chart of prediction error quantification procedure

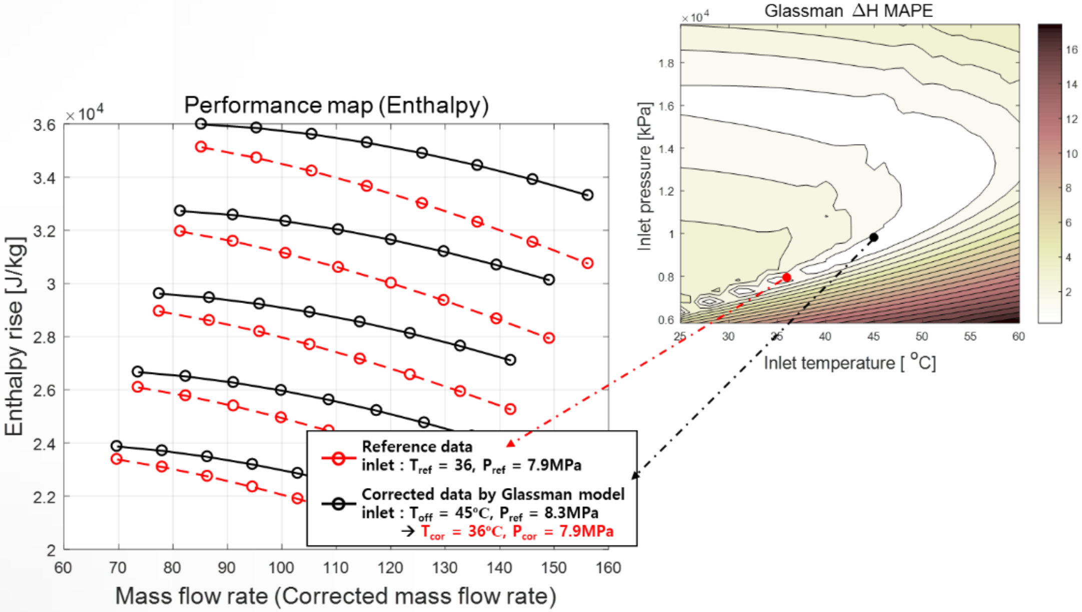


Fig. Example of error contour and comparison between reference and corrected performance data

1. Generate reference performance data with respect to mass flow rates and rpms (Designed T, P)
2. Prescribe $T_{off}$ & $P_{off}$ range, and choose one similitude model
3. Select one off-design operating condition, and calculate its performance
4. Convert the off-design performance into corrected performance with the selected model (One to one matching of mass flow rate and rpm)
5. The performance of reference data and corrected data should be the same. The discrepancy between the two data sets are quantified as mean absolute percent error (MAPE)

**MAPE** $= \frac{100}{N} * \Sigma\left|\frac{A_t - F_t}{A_t}\right|$, $A_t$ : Real value, $F_t$ : Estimation

6. Repeat the same procedure for other off-design conditions and similitude models

Due to the lack of available data, KAIST-TMD was used to produce the compressor performance data

# S-$CO_2$ Compressor Results

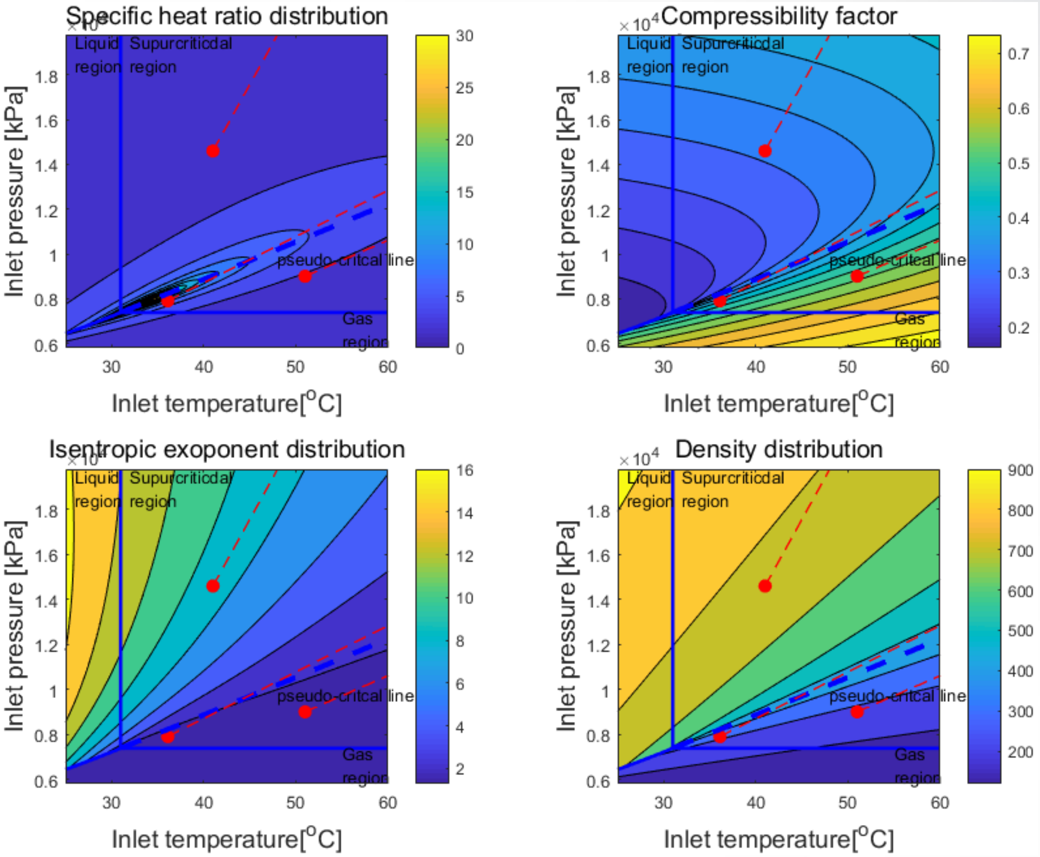
## Compressor design summary and studied range

| Design point (Comp1) | | | |
|---|---|---|---|
| $T_{in}$(°C) | 36.1 | ρ(kg/m$^3$) | 329.4 |
| $P_{in}/P_{out}$(MPa) | 7.9/20 | γ | 9 |
| m(kg/s) | 129.2 | Z | 0.41 |
| rpm(rev/min) | 15000 | $n_s$ | 1.54 |
| Efficiency(%) | 78.7 | | |

| Design point (Comp2) | | | |
|---|---|---|---|
| $T_{in}$(°C) | 51 | ρ(kg/m$^3$) | 276.7 |
| $P_{in}/P_{out}$(MPa) | 9/20 | γ | 3.6 |
| m(kg/s) | 129.2 | Z | 0.53 |
| rpm(rev/min) | 15000 | $n_s$ | 1.49 |
| Efficiency(%) | 73.8 | | |

| Design point (Comp3) | | | |
|---|---|---|---|
| $T_{in}$(°C) | 41 | ρ(kg/m$^3$) | 767.3 |
| $P_{in}/P_{out}$(MPa) | 14.6/25 | γ | 2.9 |
| m(kg/s) | 129.2 | Z | 0.32 |
| rpm(rev/min) | 15000 | $n_s$ | 8.9 |
| Efficiency(%) | 82 | | |

| | Min | Max | Resolution |
|---|---|---|---|
| T(°C) | 25 | 60 | 36 |
| P(MPa) | 5.8 | 19.8 | 29 |
| m(kg/s) | 77.5 | 142.1 | 8 |
| rpm(rev/min) | 13500 | 16500 | 5 |



- Comp1, 2, 3 were designed near the critical point, in the gas-like and liquid-like supercritical regions, respectively

- Thermodynamic properties show dramatic changes over the studied region

# S-CO$_2$ Compressor Results

## Average prediction errors

| | PR MAPE [%] | | |
|---|---|---|---|
| | Comp1 | Comp2 | Comp3 |
| **IG** | 41.2 | 30.1 | 24.5 |
| **IGZ** | 43.9 | 21.5 | 18.0 |
| **Glassman** | 18.0 | 19.3 | 14.5 |
| **BNI** | 18.0 | 11.2 | 5.9 |
| **CEA** | 119.5 | 100.1 | 21.0 |

Table. Pressure ratio average prediction error [%]

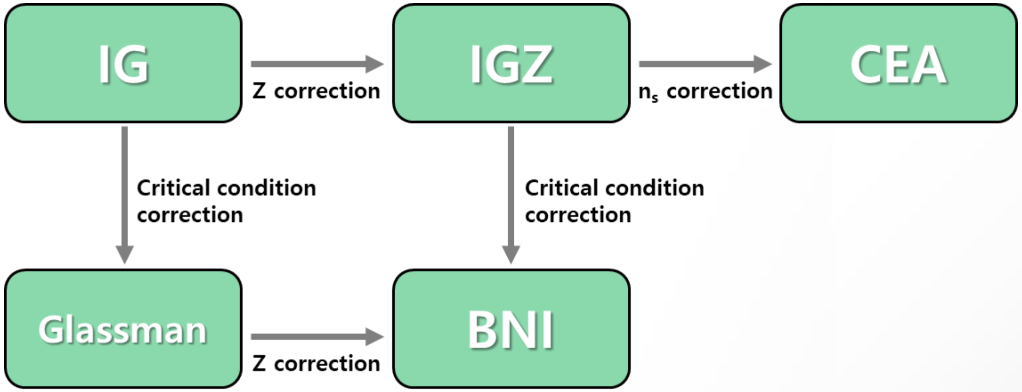
| | $\Delta$H MAPE [%] | | |
|---|---|---|---|
| | Comp1 | Comp2 | Comp3 |
| **IG** | 8.2 | 2.2 | 12.7 |
| **IGZ** | 7.6 | 3.8 | 3.6 |
| **Glassman** | 3.3 | 1.5 | 12.5 |
| **BNI** | 3.5 | 3.5 | 3.0 |
| **CEA** | 1.0 | 0.46 | 0.17 |

Table. Enthalpy rise average prediction error [%]

| | Eff MAPE [%] | | |
|---|---|---|---|
| | Comp1 | Comp2 | Comp3 |
| **IG** | 8.2 | 17.0 | 4.2 |
| **IGZ** | 8.3 | 16.2 | 1.8 |
| **Glassman** | 7.5 | 14.9 | 3.8 |
| **BNI** | 7.2 | 16.6 | 1.6 |
| **CEA** | 7.8 | 12.0 | 0.49 |

Table. Efficiency average prediction error [%]

- In contrast to air compressor results, enthalpy rise and pressure ratio prediction errors show different accuracies
- Enthalpy rise and pressure ratio are interchangeable expression of performance indicator, but their performance predictions do not accord with interchangeable character
- Enthalpy rise prediction outperforms pressure ratio prediction, and the accuracy of CEA model is comparable to air cases
- Efficiency predictions show little differences between the models (Comp1, Comp2)
- Efficiency predictions of Comp3 show less than 5% errors



IG

Z correction

IGZ

n$_s$ correction

CEA

Critical condition correction

Critical condition correction

Glassman

Z correction

BNI

Fig. Summary of model development

# Pressure Ratio / Enthalpy Rise

| | Derived PR MAPE | | |
|---|---|---|---|
| | Comp1 | Comp2 | Comp3 |
| IG | 5.8 | 1.4 | 5.5 |
| IGZ | 5.5 | 2.5 | 1.6 |
| Glassman | 2.3 | 1.0 | 5.4 |
| BNI | 2.5 | 2.4 | 1.3 |
| CEA | 0.72 | 0.30 | 0.076 |

Table. Derived pressure ratio average prediction error [%]

| | PR MAPE | | |
|---|---|---|---|
| | Comp1 | Comp2 | Comp3 |
| IG | 41.2 | 30.1 | 24.5 |
| IGZ | 43.9 | 21.5 | 18.0 |
| Glassman | 18.0 | 19.3 | 14.5 |
| BNI | 18.0 | 11.2 | 5.9 |
| CEA | 119.5 | 100.1 | 21.0 |

Table. Pressure ratio average prediction error [%]

Similitude for ΔH

$$\left(\frac{\Delta H}{\gamma RT}\right)_{cor} = \left(\frac{\Delta H}{\gamma RT}\right)_{off}$$

$$\Delta H_{cor} = (\gamma RT)_{cor}\left(\frac{\Delta H}{\gamma RT}\right)_{off}$$

$$H_{out,isen} = H_{in} + \Delta H_{cor}$$

$$P_{out} = fn(S_{in}, H_{out,isen})$$

$$PR = P_{out}/P_{in}$$

Similitude for PR

$$PR_{cor} = PR_{off}$$

**Not the same**

In case of ideal gas

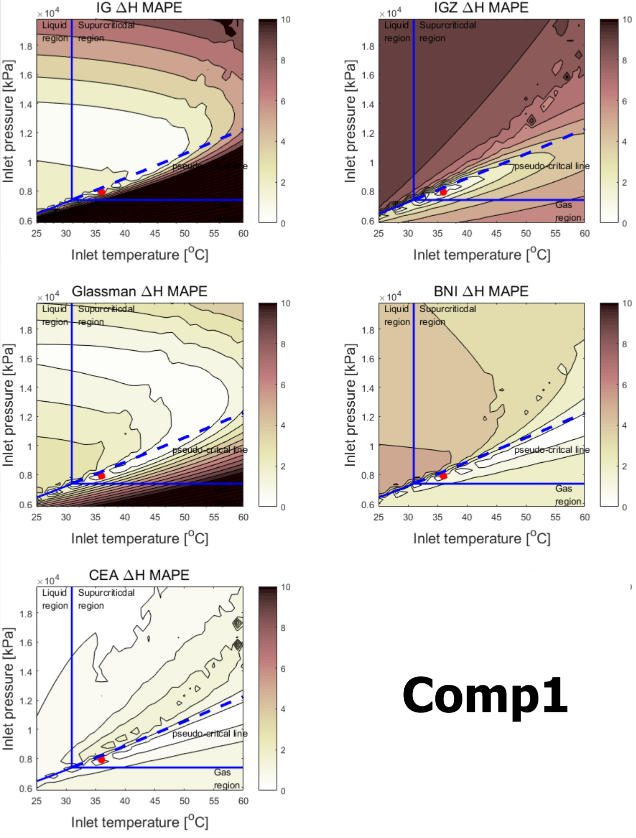
$$\frac{T_{out}}{T_{in}} = \left(\frac{P_{out}}{P_{in}}\right)^{(\gamma-1)/\gamma}, C_P = \frac{\gamma R}{\gamma-1}$$

$$\Delta H = H_{out} - H_{in} = C_P(T_{out} - T_{in}) = C_P T_{in}\left(\frac{T_{out}}{T_{in}} - 1\right) = \frac{\gamma}{\gamma-1}RT_{in}\left(\left(\frac{P_{out}}{P_{in}}\right)^{(\gamma-1)/\gamma} - 1\right)$$

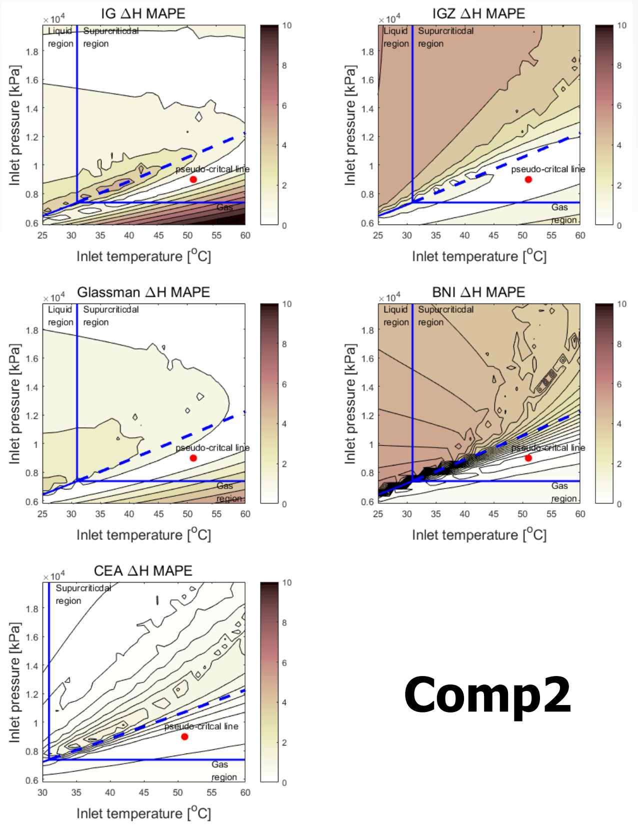
$$\frac{\Delta H}{\gamma RT_{in}} = \frac{1}{\gamma-1}\left(\left(\frac{P_{out}}{P_{in}}\right)^{(\gamma-1)/\gamma} - 1\right) = fn(P_{out}/P_{in})$$

- Enthalpy rise and pressure ratio are interchangeable expression, but their similitude are not the same
- The property changes S-$CO_2$ make it hard to relate these two similitudes
- Overall, enthalpy rise predictions have better accuracy than pressure ratio predictions
- At this point, CEA model is the best model to predict enthalpy rise

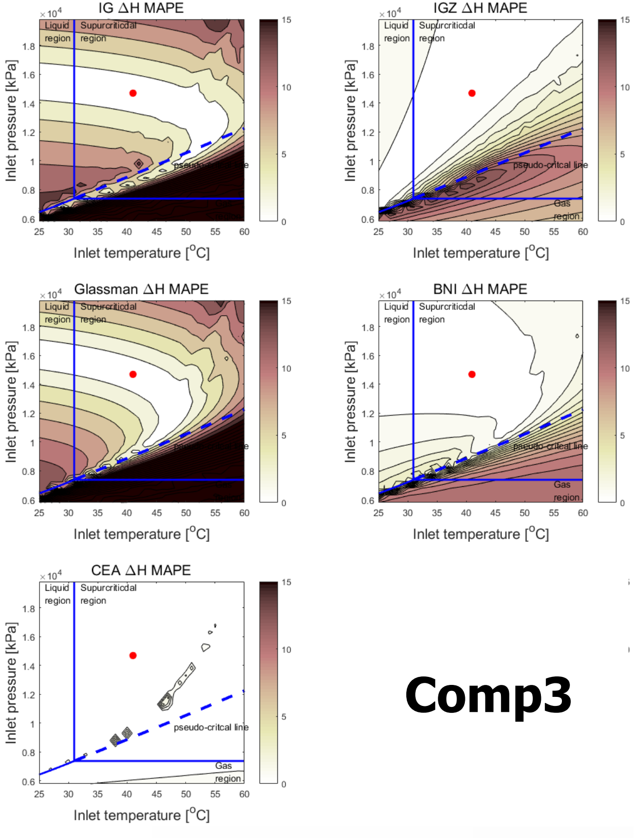
# Enthalpy Rise Prediction



Comp1



Comp2

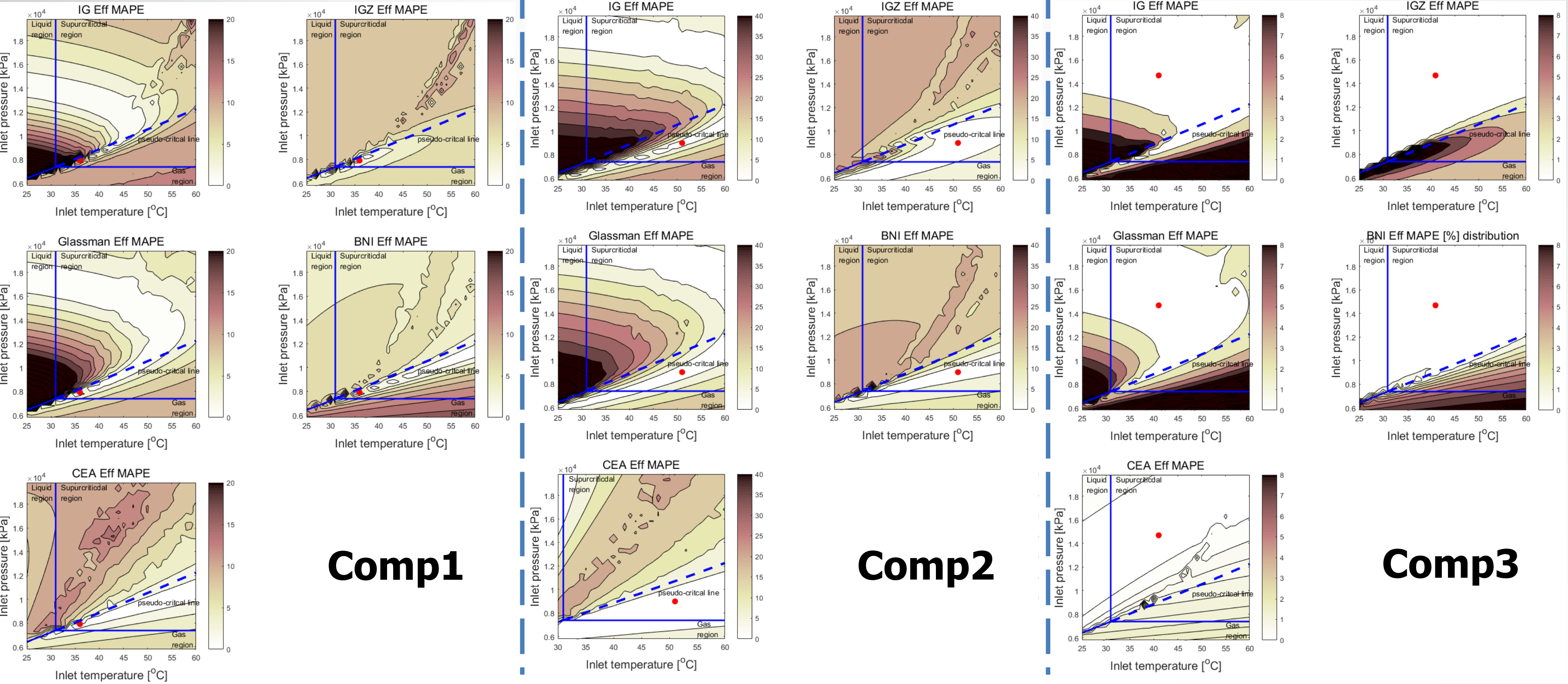


Comp3

| Enthalpy rise prediction | | MAPE [%] | | |
|---|---|---|---|---|
| | | Comp1 | Comp2 | Comp3 |
| IG | Max | 41.5 | 12.8 | 57.2 |
| | Min | 0.41 | 0.032 | 0.08 |
| IGZ | Max | 10.6 | 6.0 | 13.7 |
| | Min | 0.06 | 0.014 | 0.009 |
| Glassman | Max | 18.3 | 6.7 | 52.1 |
| | Min | 0.19 | 0.011 | 0.05 |
| BNI | Max | 6.10 | 6.2 | 11.2 |
| | Min | 0.038 | 0.009 | 0.016 |
| CEA | Max | 3.66 | 1.7 | 2.4 |
| | Min | 0.027 | 0.002 | 0.002 |

Table. Summary of max and min of prediction error [%]

- Besides average errors, maximum errors needs to be checked
- Each models has different shapes of error distribution
- The models tend to work well in the region where a compressor is designed after the corrections
- CEA model shows sufficient maximum error as well



| Efficiency prediction | | MAPE [%] | | |
|---|---|---|---|---|
| | | Comp1 | Comp2 | Comp3 |
| IG | Max | 41.2 | 59.4 | 28.3 |
| | Min | 0.78 | 0.082 | 0.018 |
| IGZ | Max | 17.4 | 30.1 | 27.7 |
| | Min | 0.55 | 0.082 | 0.003 |
| Glassman | Max | 46.6 | 56.8 | 26.0 |
| | Min | 0.40 | 0.16 | 0.038 |
| BNI | Max | 18.7 | 48.0 | 8.9 |
| | Min | 0.069 | 0.12 | 0.008 |
| CEA | Max | 16.6 | 27.5 | 2.0 |
| | Min | 0.092 | 0.011 | 0.003 |

Table. Summary of max and min of prediction error [%]

- Efficiency predictions have larger errors than enthalpy rise predictions
- CEA model has about 10% average errors in Comp1 and Comp2, but 16.6%, 27.5% maximum errors are observed locally
- Although CEA model has a good performance to predict enthalpy rise, efficiency prediction needs to be improved

# Efficiency Prediction

# Off-design Prediction Model Improvement

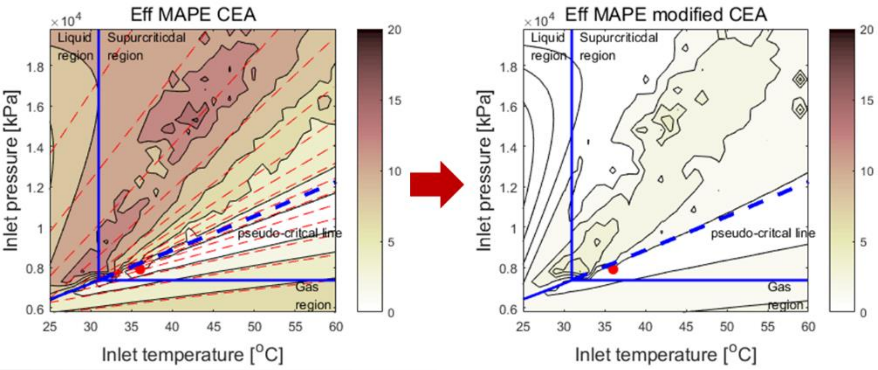


Fig. Efficiency correction by density in case of n=0.23 (Comp1)

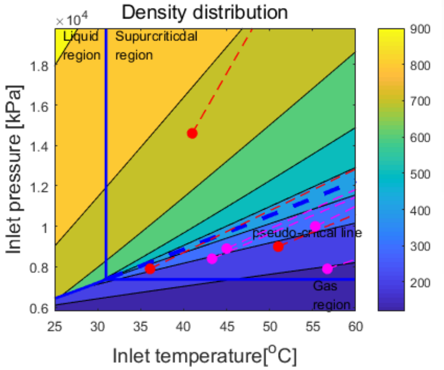


Fig. Density contour with design point of compressors

- There are researches which tried to improve efficiency prediction

Glassman, $\frac{1-\eta_A}{1-\eta_B} = \left(\frac{Re_B}{Re_A}\right)^n$, n=0.1~0.2 ; Re effect correction

Roberts, $\frac{1-\eta_A}{1-\eta_B} = \left(\frac{\gamma_A}{\gamma_B}\right)^n$, n=0.8 ; γ effect correction

$\frac{1-\eta_{ref}}{1-\eta_{off}} = \left(\frac{\rho_{off}}{\rho_{ref}}\right)^n$, n=? ; Density effect correction

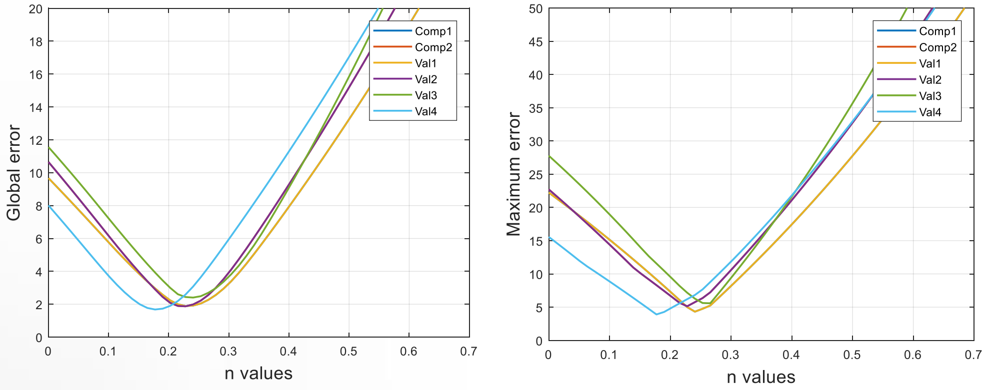


Fig. Global and maximum error variation according to exponent n

| MAPE | Average error | | | Maximum error | | |
|---|---|---|---|---|---|---|
| Prediction | Enthalpy rise | Efficiency | | Enthalpy rise | Efficiency | |
| | CEA (Modified CEA) | CEA | Modified CEA | CEA (Modified CEA) | CEA | Modified CEA |
| **Comp1** | 1.0 | 7.6 | **1.6** | 3.7 | 16.6 | **5.0** |
| **Comp2** | 0.45 | 12.0 | **2.18** | 1.7 | 27.5 | **5.8** |
| **Val1** | 0.56 | 9.7 | **1.88** | 2.1 | 22.2 | **4.9** |
| **Val2** | 0.44 | 10.6 | **1.88** | 1.5 | 22.7 | **5.2** |
| **Val3** | 0.61 | 11.6 | **2.42** | 2.6 | 27.8 | **7.0** |
| **Val4** | 0.50 | 8.0 | **2.66** | 1.2 | 15.6 | **6.3** |

# Summary & Conclusion

1. To predict off-design performance of compressor, the concept of “corrected mass flow rate and rpm” has been used for an air compressor. However, the validity of such methods needs to be evaluated for an S-$CO_2$ compressor

2. Five existing correction models were collected from the open literatures, the average and maximum prediction errors were evaluated according to the established error quantification and visualization procedure

3. In contrast to an air compressor, it is recommended to use enthalpy rise for similitude models, because constant property assumption does not work in S-$CO_2$

4. The similitude tends to work well in the region where a compressor is designed, which means the compressor designed in the gas-like supercritical region has high prediction accuracy in the gas-like region and vice versa

5. CEA model is the most suitable model for an S-$CO_2$ compressor. However, its efficiency prediction error needs to be refined because of the large maximum errors, especially in case of a compressor designed in the gas-like supercritical region, but operating in the liquid-like region

6. Therefore, Modified CEA model was proposed to improve efficiency prediction

# Thank you