Effects of thermal boundary condition on turbulent statistics in flows with a supercritical fluid

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Motivation

Several numerical studies on heat transfer to supercritical fluids
- Effect of buoyancy, heat flux/mass flux ratio, etc.
- Yoo, Annual Review Fluid Mechanics, 2013

Most of the numerical studies assume isoflux boundary conditions
- Isoflux BC allows temperature fluctuations at the wall
- Isothermal BC does not allow temperature to fluctuate at the wall

If fluid’s Prandtl number > 1, temperature fluctuations do not affect heat transfer (Kasagi, 1989; Li et al., 2009).

Does this also hold for flows with strong property gradients even if Pr > 1?
Effect of fluid/wall properties on temperature fluctuations

Thermal effusivity ratio:

\[ K = \sqrt{\frac{\rho_f c_p, f \lambda_f}{\rho_s c_s \lambda_s}} \rightarrow \infty : \text{isoflux BC} \]

\[ K = \sqrt{\frac{\rho_f c_p, f \lambda_f}{\rho_s c_s \lambda_s}} \rightarrow 0 : \text{isothermal BC} \]

From Tiselj et al. 2001, JHT
Effect of Prandtl number

- Ratio of Nusselt number for isoflux to isothermal boundary conditions

Sleicher, 1955; Kasagi et al., 1989
Thermal effusivity ratio and Prandtl number examples

<table>
<thead>
<tr>
<th>Prandtl number</th>
<th>Air * 0.708</th>
<th>Water * 6.78</th>
<th>scCO$_2$ (80bar) up to 14</th>
</tr>
</thead>
<tbody>
<tr>
<td>Aluminum</td>
<td>0.00025</td>
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* based on Kasagi et al., Journal of heat transfer, 1989

Investigate influence of thermal effusivity ratio on heat transfer to scCO$_2$
- Allow wall temperature fluctuations: $K = \infty$
- Do not allow temperature fluctuations: $K = 0$
Simulation setup

This setup ensures the same thermodynamic condition at the wall!
Simulation setup

Pressure constant at P=80 bar

Isoflux boundary condition (heat flux Q)

Inflow generator
Properties of supercritical fluids

T-s Diagram including the critical point for CO₂

- P=7.8 Mpa
- P=8.0 Mpa
- P=8.2 Mpa
Governing equations

Low-Mach number approximation of Navier-Stokes equations:

\[
\begin{align*}
\frac{\partial \rho}{\partial t} + \frac{\partial \rho u_i}{\partial x_i} &= 0 \\
\frac{\partial \rho u_i}{\partial t} + \frac{\partial \rho u_i u_j}{\partial x_j} &= -\frac{\partial p}{\partial x_i} + \frac{1}{Re_{\tau_0}} \frac{\partial \tau_{ij}}{\partial x_j} \\
\frac{\partial \rho h}{\partial t} + \frac{\partial \rho u_i h}{\partial x_i} &= -\frac{1}{Re_{\tau_0} Pr_0} \frac{\partial q_i}{\partial x_i}
\end{align*}
\]

\[
\tau_{ij} = \mu S_{ij} = \mu \left( \frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i} - \frac{2}{3} \frac{\partial u_k}{\partial x_k} \delta_{ij} \right)
\]

\[
q_i = -\lambda \frac{\partial T}{\partial x_i} = -\frac{\lambda}{c_p} \frac{\partial h}{\partial x_i} = -\alpha \frac{\partial h}{\partial x_i}
\]

with:

\[Re_{\tau_0} = \frac{\rho_0^* u_{\tau_0}^* D^*}{\mu_0^*} = 360\]

\[Pr_0 = \frac{\mu_0^* c_p^*}{\lambda_0^*} = 3.2\]

\[Q = \frac{q_w^* D^*}{\lambda_0^* T_0^*} = Re_{\tau_0} Pr q = 2.4\]
Numerical scheme

- **Spatial discretization**: 2\textsuperscript{nd} order central difference on staggered mesh
- **Temporal discretization**: 2\textsuperscript{nd} Adams-Bashforth and Adams-Moulton
- Koren limiter for advection part of energy equation
- Diffusion part in circumferential direction treated implicitly
- Mesh resolution:
  - Mesh points 128 x 288 x 1728
  - Radial 0.55 (wall) < $\Delta r^+$ < 4.3 (center)
  - Circumferential $R \Delta \theta^+ = 3.93$
  - Axial $\Delta z^+ = 6.25$
- **Thermophysical properties** (**CO\textsubscript{2} at P=8 Mpa**) are interpolated from table
Instantaneous flow field, isoflux simulation

Stream-wise velocity

Enthalpy
Instantaneous enthalpy fluctuations

\[ y^+ = 4.7 \text{ (based on inlet condition)} \]
Enthalpy rms profiles

\[ x/D = 15 \]

\[ K = \infty \]

\[ K = 0 \]
Radial heat fluxes

Total radial heat flux:

\[ q_{r,\text{tot}} = \overline{\alpha} \frac{\partial \bar{h}}{\partial r} + \alpha' \frac{\partial \bar{h}'}{\partial r} - \rho u'' h'' \]

Additional heat flux caused by:

\[ \alpha' \frac{\partial \bar{h}'}{\partial r} \]

\[ x/D = 15 \]

\[ K = \infty \]

\[ K = 0 \]
Nusselt number ratio

Nusselt number:

\[ \frac{N_{u_{\text{isoflux}}}}{N_{u_{\text{isothermal}}}} = \frac{\alpha \frac{\partial h}{\partial r}|_w}{\lambda_b(T_w - T_b)} \]

Supercritical CO\textsubscript{2}

Constant property fluid Pr=3.2

7% higher Nu number
Turbulent kinetic energy and Reynolds shear stress

Turbulent kinetic energy

Reynolds shear stress

\[ x/D = 15 \]

\[ y = 1 - 2r \]

\[ k = \frac{1}{2} \rho u_i' u_i' \]

\[ \frac{\varepsilon}{ho u_i' u_j'} \]

- \[ K = \infty \]
- \[ K = 0 \]
Decomposed skin friction, FIK identity

(Fukagata, Iwamoto, Kasagi; PoF 2002)

\[
C_{f,FIK} = -\frac{2}{\rho_b U_b^2 R e_0} \int_0^R r \mu S_{rz} r dr + \frac{2}{\rho_b U_b^2} \int_0^R r \rho U_z \frac{\partial \tilde{u} U_z}{\partial r} r dr + \frac{1}{\rho_b U_b^2} \int_0^R (R^2 - r^2) \frac{\partial \tilde{U}_r}{\partial r} r dr + \frac{1}{\rho_b U_b^2} \int_0^R (R^2 - r^2) \frac{\partial \tilde{u}_r U_z}{\partial r} r dr
\]

\[
+ \frac{1}{\rho_b U_b^2} \int_0^R (R^2 - r^2) \frac{\partial \tilde{u}_r U_z}{\partial r} r dr + \frac{1}{\rho_b U_b^2} \int_0^R (R^2 - r^2) \frac{\partial \tilde{u}_r U_z}{\partial r} r dr - \frac{1}{\rho_b U_b^2 R e_0} \int_0^R (R^2 - r^2) \frac{\partial \mu' S_{zz}'}{\partial r} r dr
\]

\[= \Phi(r, z) = \Phi(r, z) - 8 \int_0^R \Phi(r, z) r dr \]

Laminar contribution

Turbulent contribution

Inhomogeneous contribution

Fully developed pipe flow with constant property fluid

\[
16 \frac{1}{Re_b}
\]
Decomposed skin friction, FIK identity

Total skin friction
Laminar contribution
Turbulent contribution
Inhomogeneous contribution

Dashed lines: Iso-flux
Symbols: Iso-thermal
Decomposed Nusselt number, FIK identity

\[
N_{u_{FIK}} = \frac{32}{\lambda_b(T_w - T_b)} \int_0^R r \frac{\partial \bar{h}}{\partial r} r dr - \frac{32 \text{Re}_0 \text{Pr}_0}{\lambda_b(T_w - T_b)} \int_0^R r \rho \theta'' u'' r dr + \frac{16 \text{Re}_0 \text{Pr}_0}{\lambda_b(T_w - T_b)} \int_0^R (R^2 - r^2) \frac{1}{r} \frac{\partial \bar{h} \theta'}{\partial r} r dr

- \frac{16 \text{Re}_0 \text{Pr}_0}{\lambda_b(T_w - T_b)} \int_0^R (R^2 - r^2) \frac{\partial \bar{h} \theta}{\partial z} r dr - \frac{16 \text{Re}_0 \text{Pr}_0}{\lambda_b(T_w - T_b)} \int_0^R (R^2 - r^2) \frac{\partial \bar{h} \theta'}{\partial z} r dr + \frac{16}{\lambda_b(T_w - T_b)} \int_0^R (R^2 - r^2) \frac{\partial \bar{h} \theta}{\partial z} r dr

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Laminar contribution

Turbulent contribution

Inhomogeneous contribution
Decomposed Nusselt number, FIK identity

Total Nusselt number
Laminar contribution
Turbulent contribution
Inhomogeneous contribution

Dashed lines: Iso-flux
Symbols: Iso-thermal
Conclusions

- Thermal effusivity ratio has an effect on heat transfer even for $Pr > 1$ in supercritical flows
- Nusselt number 7% higher for $K = \infty$
- The turbulent heat flux and Reynolds shear stress decrease
- Higher enthalpy fluctuations for $K = \infty$ induce higher density fluctuations, which result in larger velocity fluctuations and thus higher mixing
Thermal activity ratio and Prandtl number examples

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Prandtl number for CO₂ at 80 bar
Temperature rms values for constant properties

Reynolds decomposition of wall heat flux:

\[ Q_w = \frac{1}{\alpha} \frac{\partial \bar{h}}{\partial r} \bigg|_w + \alpha' \frac{\partial h}{\partial r} \bigg|_w + \frac{\alpha}{\alpha} \frac{\partial h'}{\partial r} \bigg|_w \rightarrow \frac{\partial \bar{h} r^2}{\partial r} = -\frac{2}{\alpha} \frac{\bar{h}'}{\alpha'} \frac{\partial \bar{h}}{\partial r} \]
Temperature rms values for constant properties

**Constant property flow (Pr=3.2)**

\[ \frac{\partial \overline{h'^2}}{\partial r} = 0 \]

\[ \overline{h'^2} = c_0 + c_2 r^2 + \ldots \]

**Supercritical fluid flow (Pr$_0$=3.2)**

\[ \frac{\partial \overline{h'^2}}{\partial r} = -\frac{2}{\alpha} \overline{h'} \frac{\partial \overline{h}}{\partial r} \]

\[ \overline{h'^2} = c_0 + c_1 r + c_2 r^2 + \ldots \]
Radial heat fluxes

Total radial heat flux:

\[ q_{r,tot} = \bar{\alpha} \frac{\partial \bar{h}}{\partial r} + \alpha' \frac{\partial \bar{h}'}{\partial r} - \rho u'' h'' \]

Constant property (CP) flow

Specified heat flux = 2.4

Iso-flux

Iso-thermal
Effect of wall thickness on temperature fluctuations

Dimensionless wall thickness:

\[ y^{++} = \sqrt{\frac{\lambda_f}{\lambda_s}} \cdot y^+ \]

From Tiselj et al. 2001, JHT