A novel approach to accurately model heat transfer to supercritical fluids

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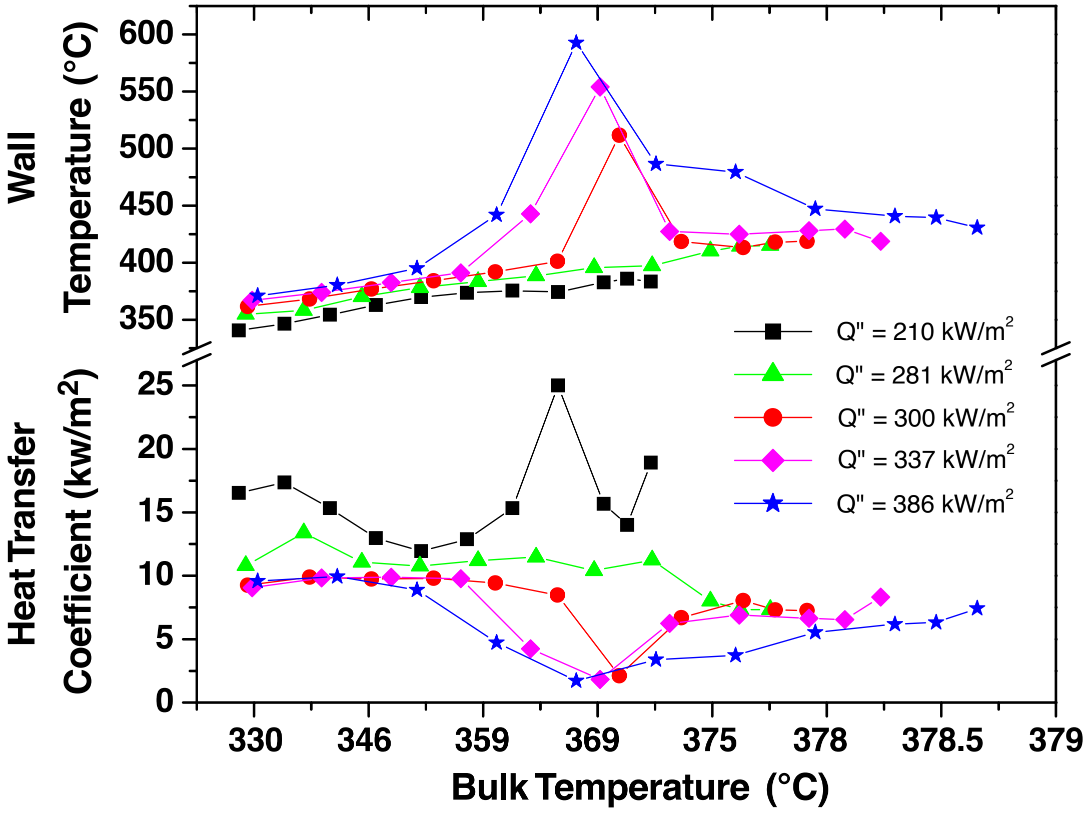
Heat transfer to supercritical fluids

authors	fluid	subjects
Dickinson (1958)	H_2O	Heat transfer
Shitsman (1959, 1963)	H_2O	Heat transfer, heat transfer deterioration, oscillation
Domin (1963)	H_2O	Heat transfer, oscillation
Bishop (1962, 1965)	H_2O	Heat transfer
Swenson (1965)	H_2O	Heat transfer, heat transfer deterioration
Ackermann (1970)	H_2O	Heat transfer, pseudo-boiling phenomena
Yamagata (1972)	H_2O	Heat transfer, heat transfer deterioration
Griem (1999)	H_2O	Heat transfer
Sabersky (1967)	CO_2	Visualisation, turbulence
Jackson (1966, 1968)	CO_2	Heat transfer, buoyancy effect
Petukhov (1979)	CO_2	Heat transfer, pressure drop
Kurganov (1985, 1993)	CO_2	Flow structure
Sakurai (2000)	CO_2	Flow visualization

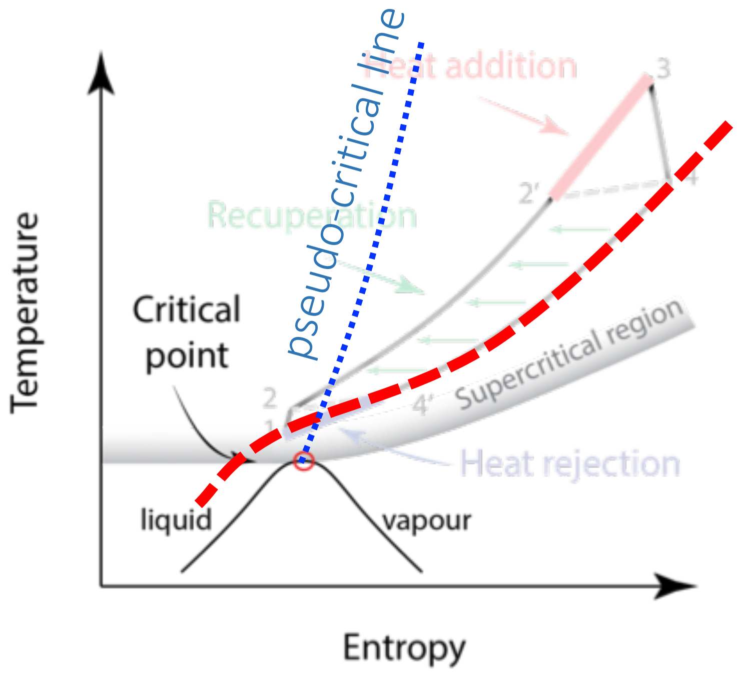
From Cheng, X., Schulenberg, T., FZKR 6609

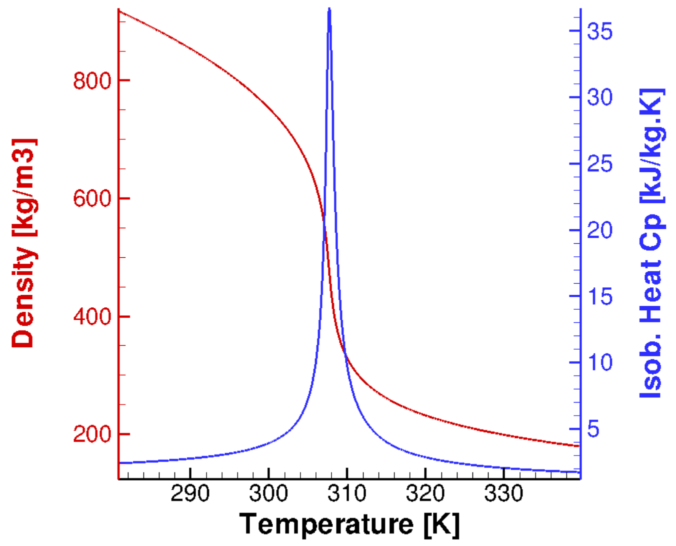
Adapted from Licht et al., Int. J. Heat and Fluid Flow, 2008

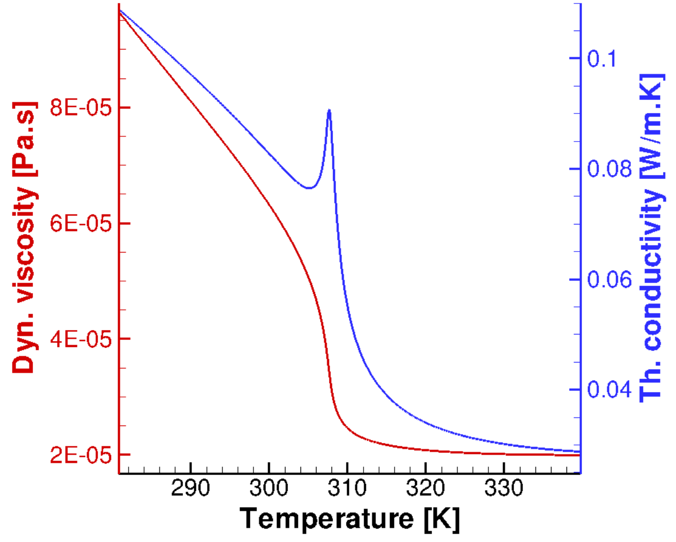


- Pressure = 233 bar
- Mass velocity = 420 kg/m²/s

Thermophysical / transport properties



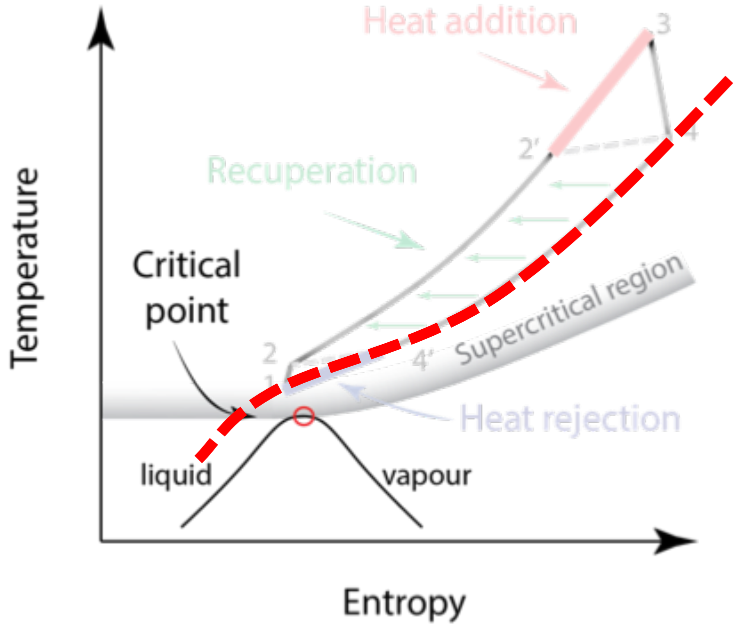


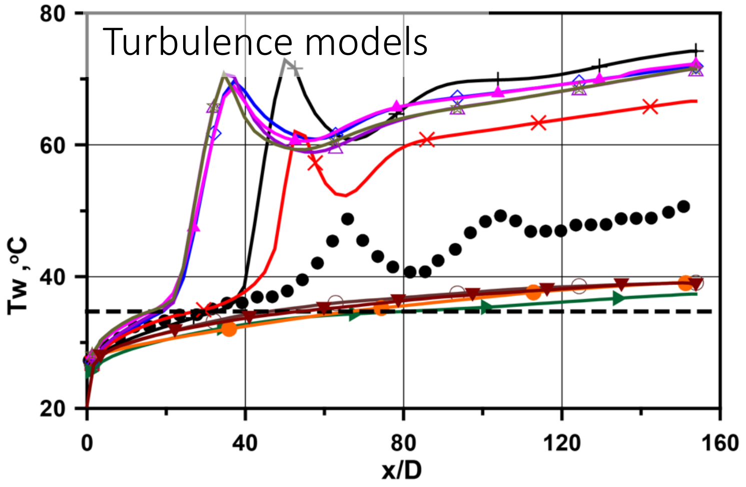


Carbon dioxide CO_2 at 80 bar

- Isobaric heat capacity max at T_{PC}
- Thermal conductivity local max at T_{PC}

Why is research needed?





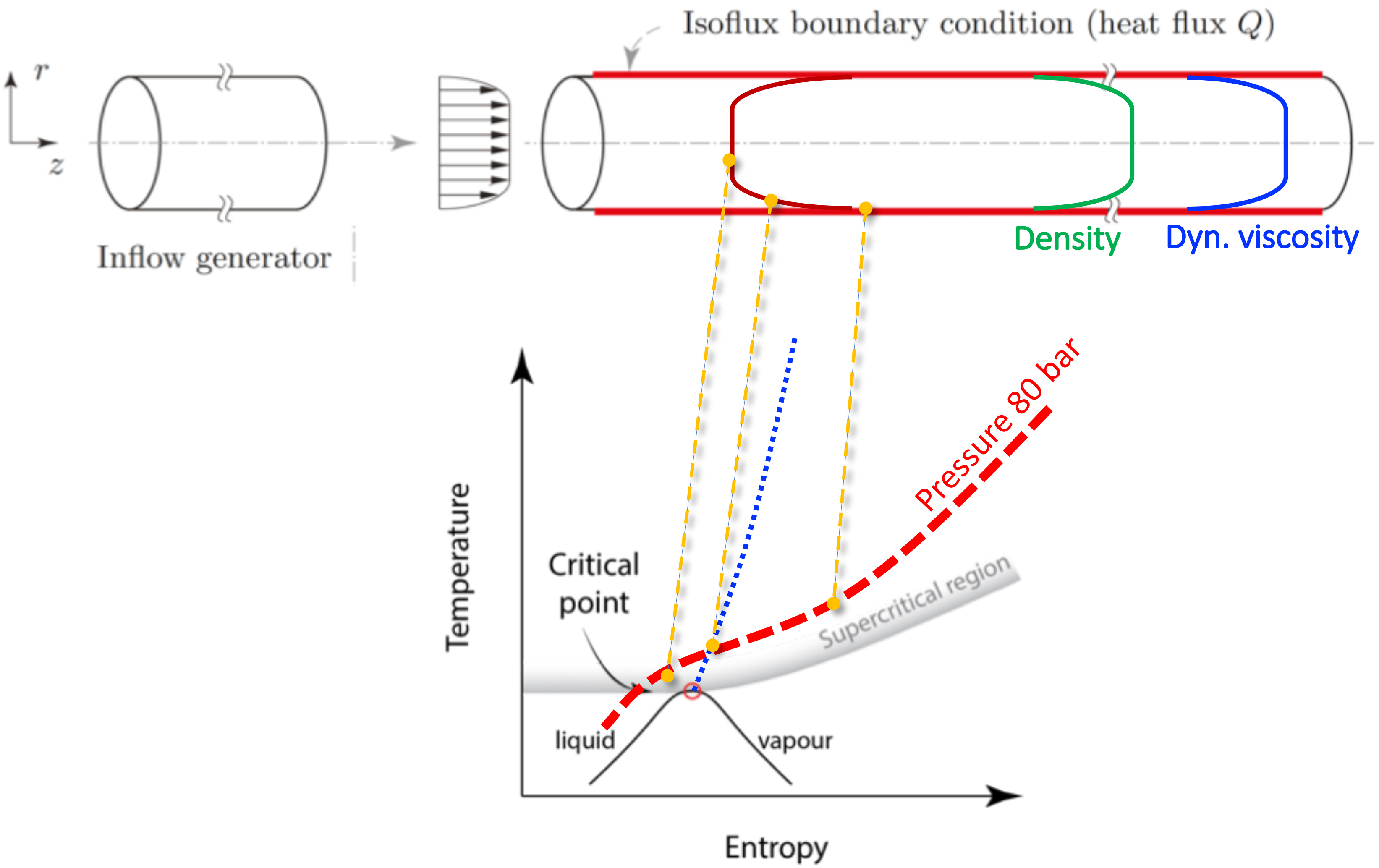
Turbulent heat transfer

Sharabi, Ambrosini, Ann. Nuclear Energy (2009)

Extreme variation of thermophysical property close to critical point

- Gas dynamics extremely complex
- Turbulence highly modified, current engineering models are not predictive

Numerical study of heat transfer using DNS



Refs.: Nemati et al., IJHMT (2015)

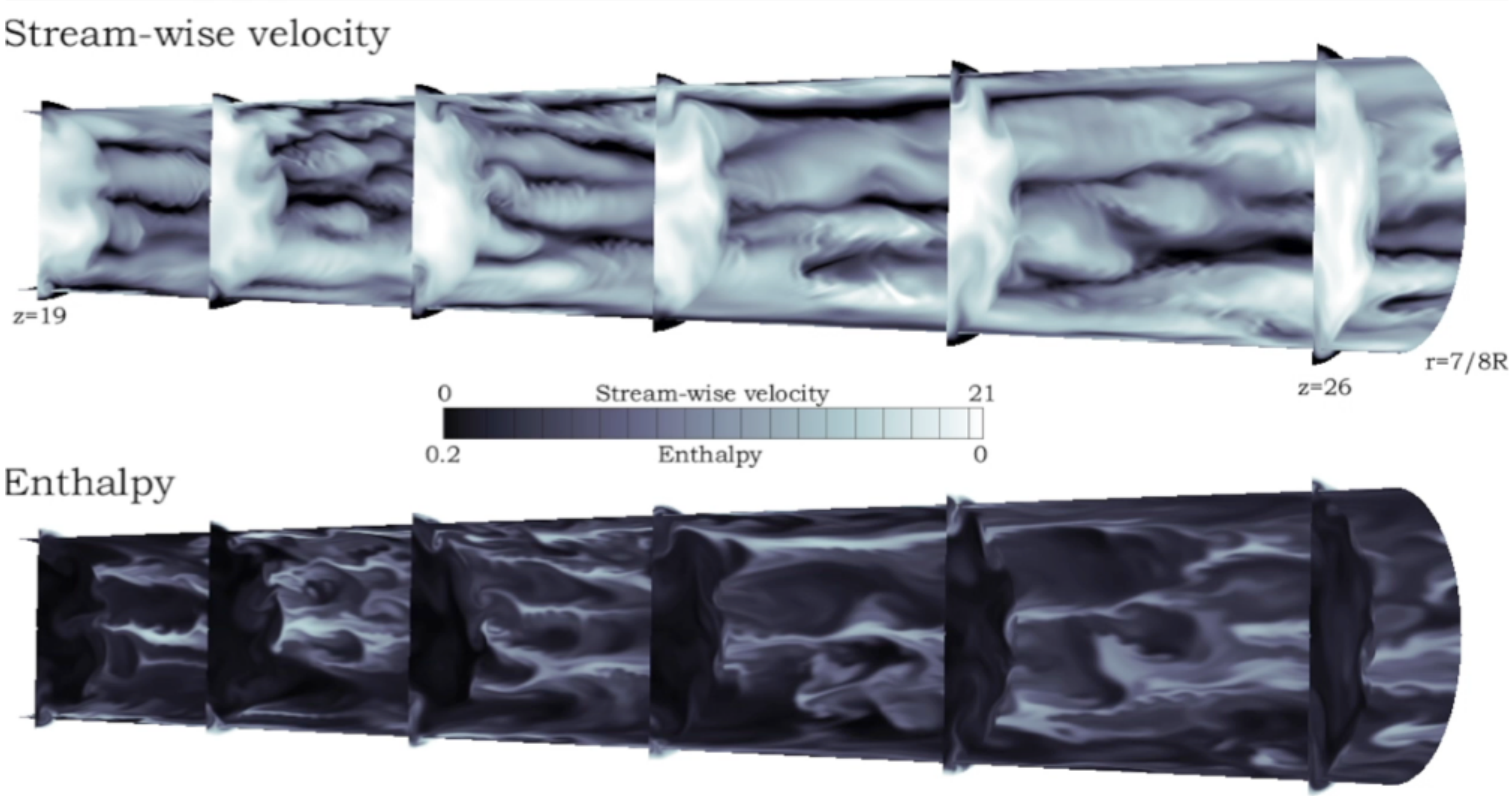
Considered cases

Case	Type	Direction / gravity	Richardson #
A	Forced	No gravity	0
B	Mixed	Upward flow ⇡	-10
C	Mixed	Upward flow ⇡	-270
D	Mixed	Downward flow ⇣	100

With:

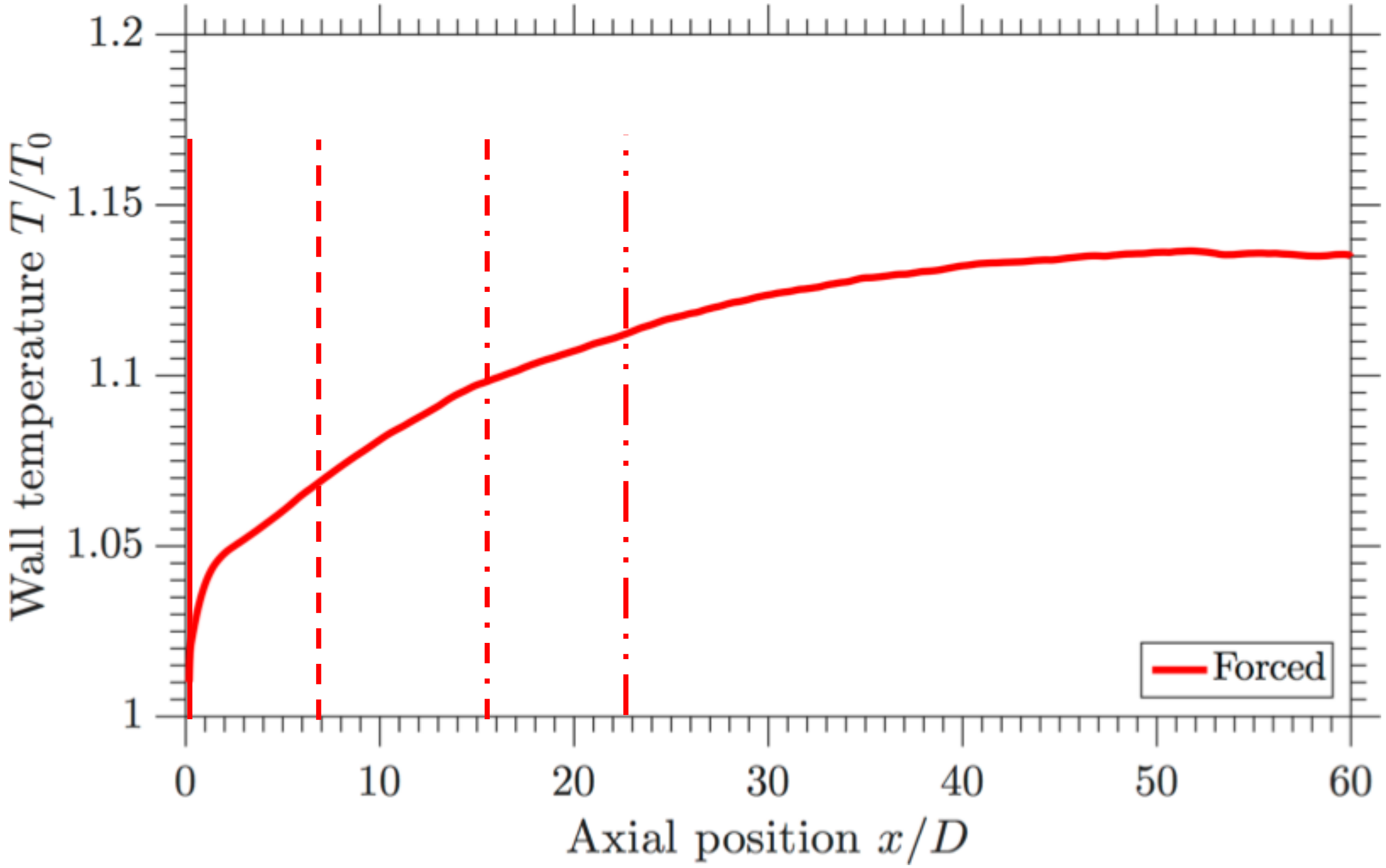
- Reynolds number: $Re_{\tau,0} = \frac{\rho_0 u_{\tau,0} D}{\mu_0} = 360$
- Prandtl number: $Pr_0 = \frac{\mu c_{p,0}}{\lambda_0} = 3.19$
- Non-dimensional heat flux: $Q = \frac{q_w D}{\lambda_0 T_0} = 2.4$

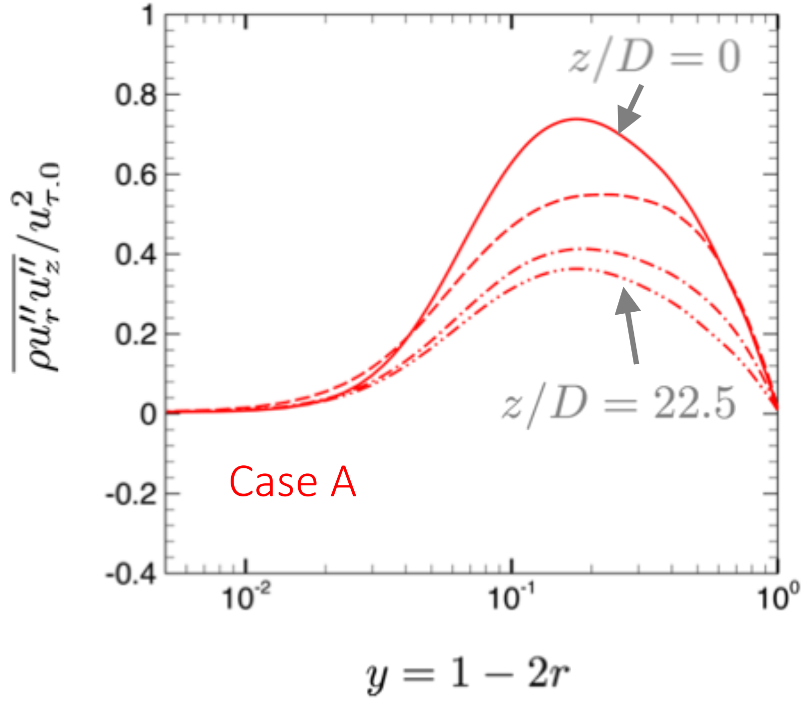
Forced convection (case A)



Nemati et al., JFM (2016)

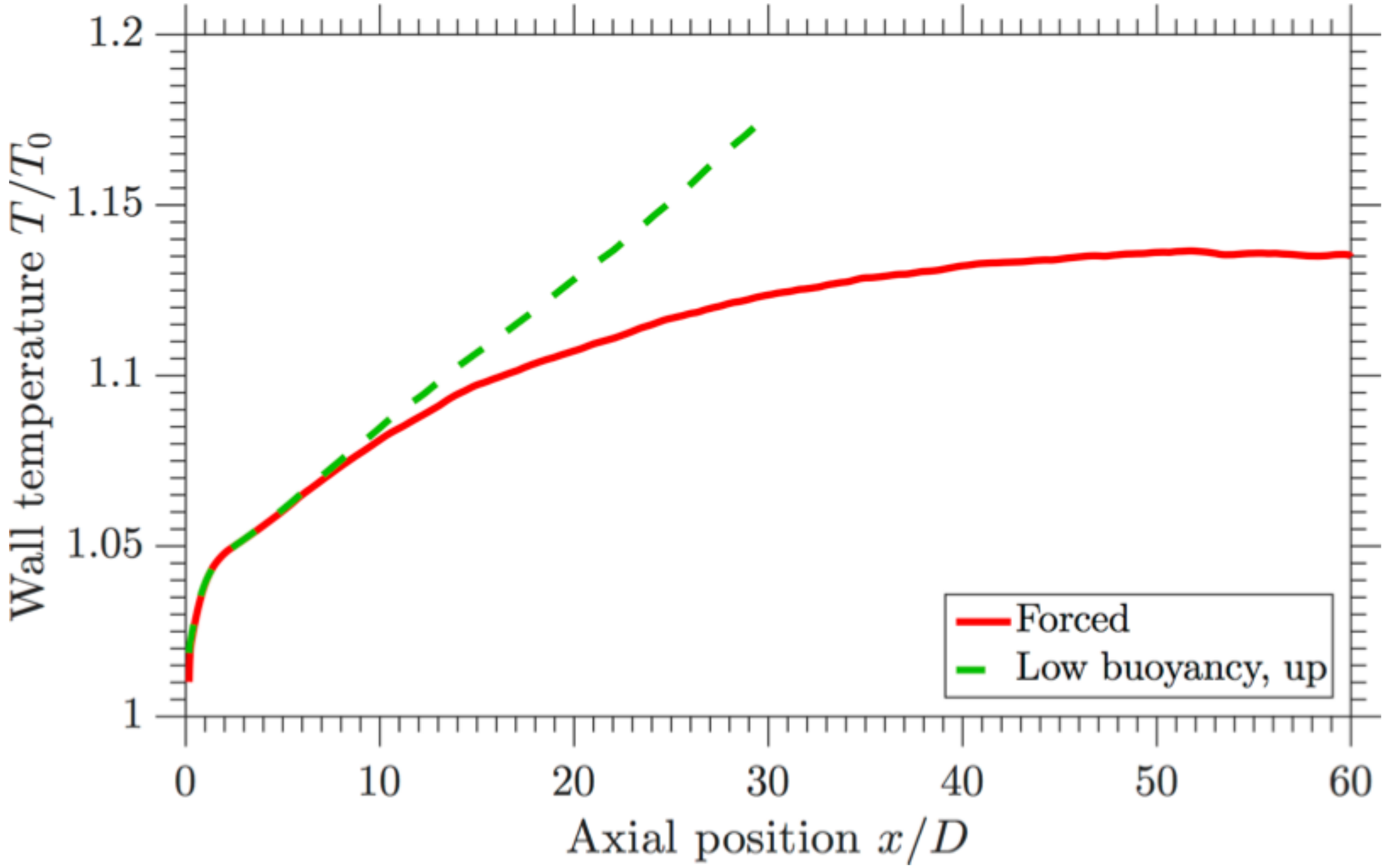
Wall temperature distribution from DNS

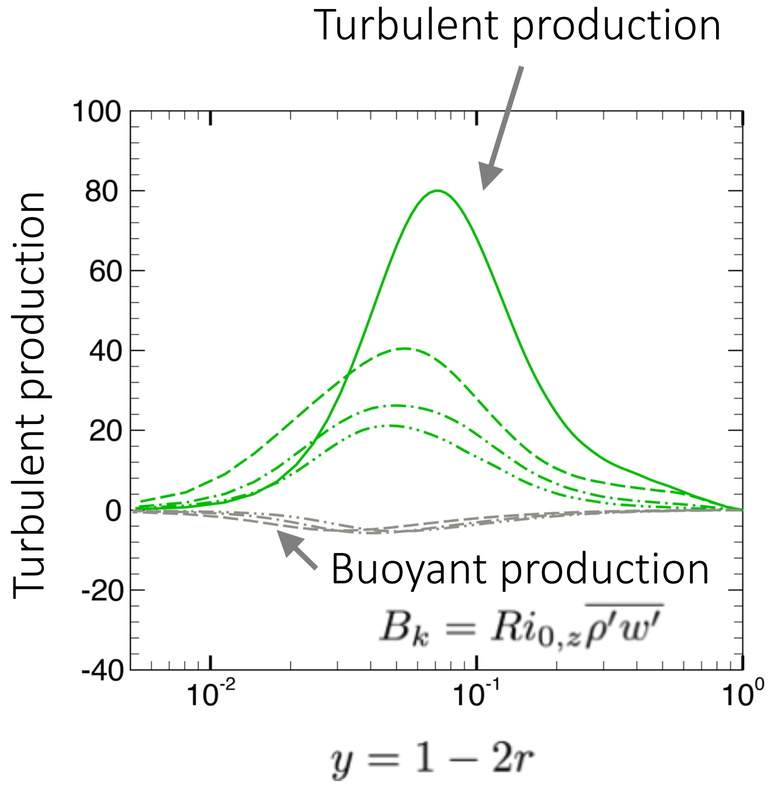




- Thermal expansion --> flow acceleration --> decrease in turbulence

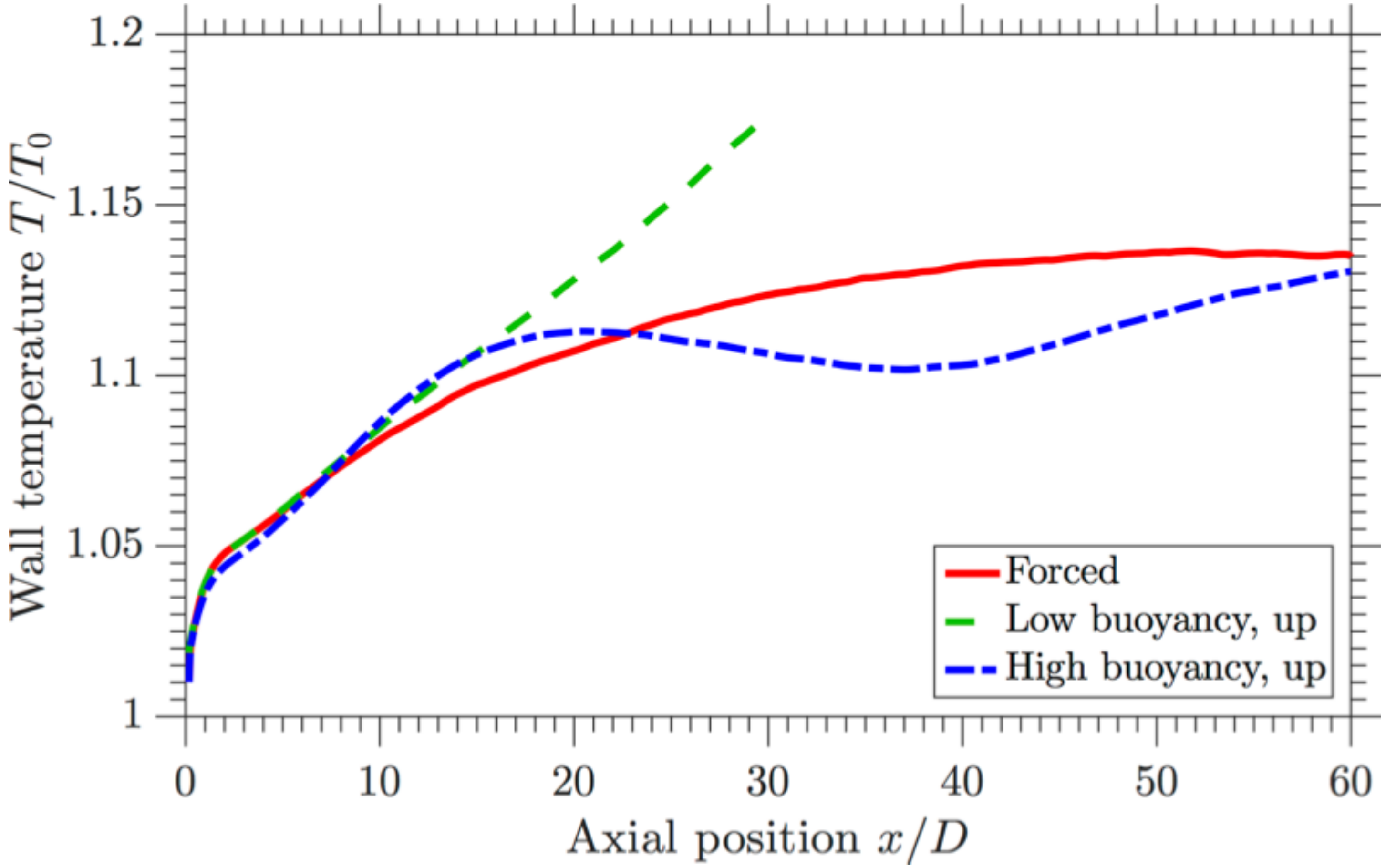
Wall temperature distribution from DNS



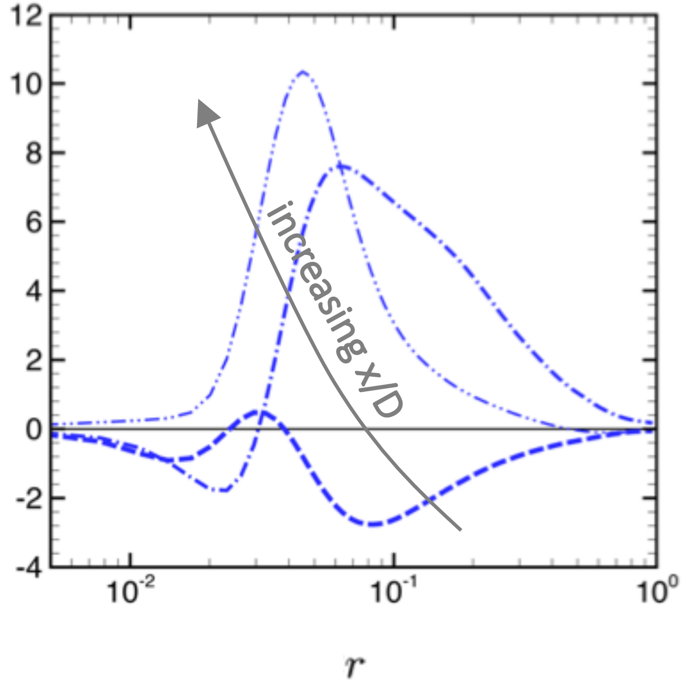


Buoyancy causes further decrease in turbulence, external effect --> further local flow acceleration

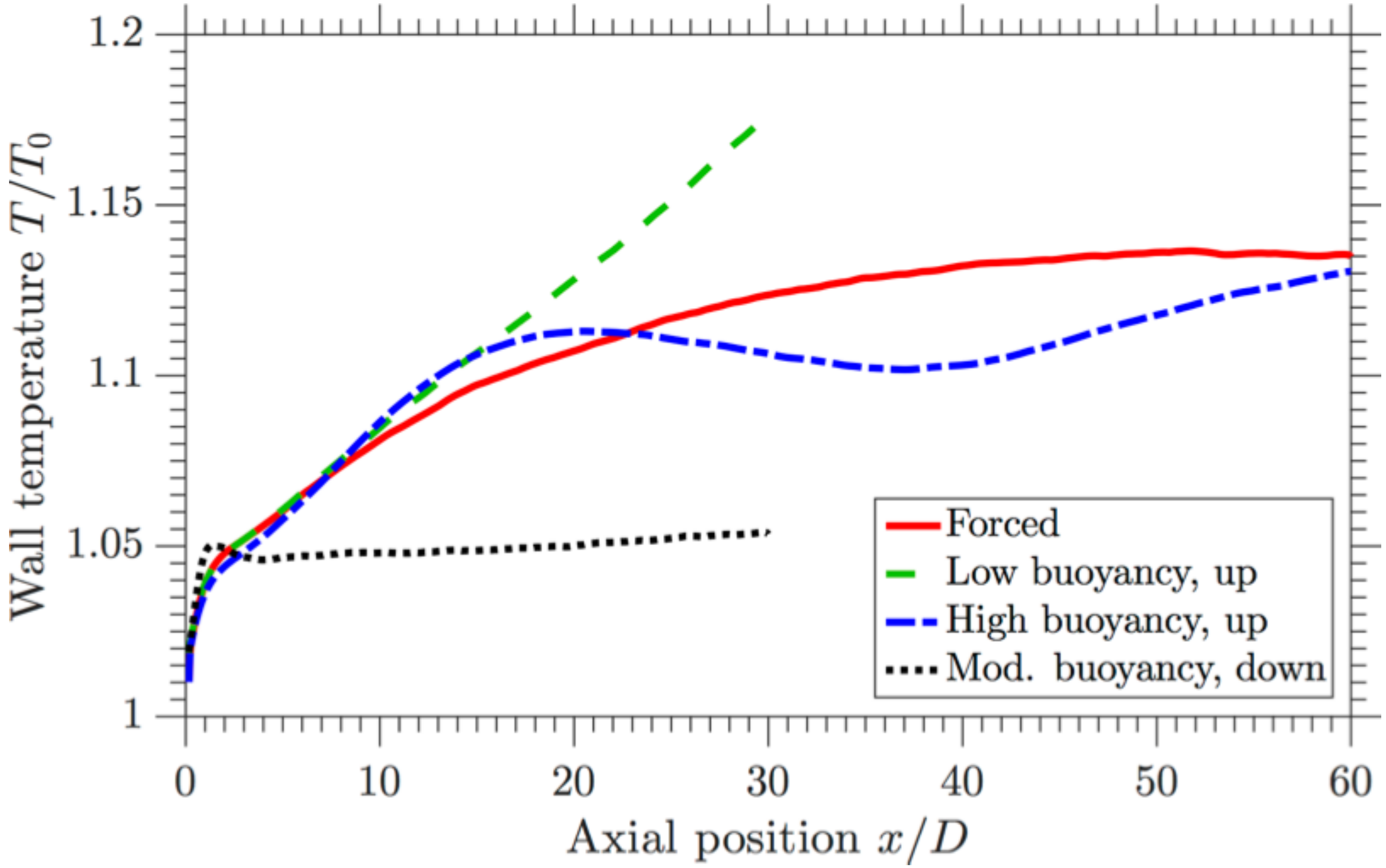
Wall temperature distribution from DNS

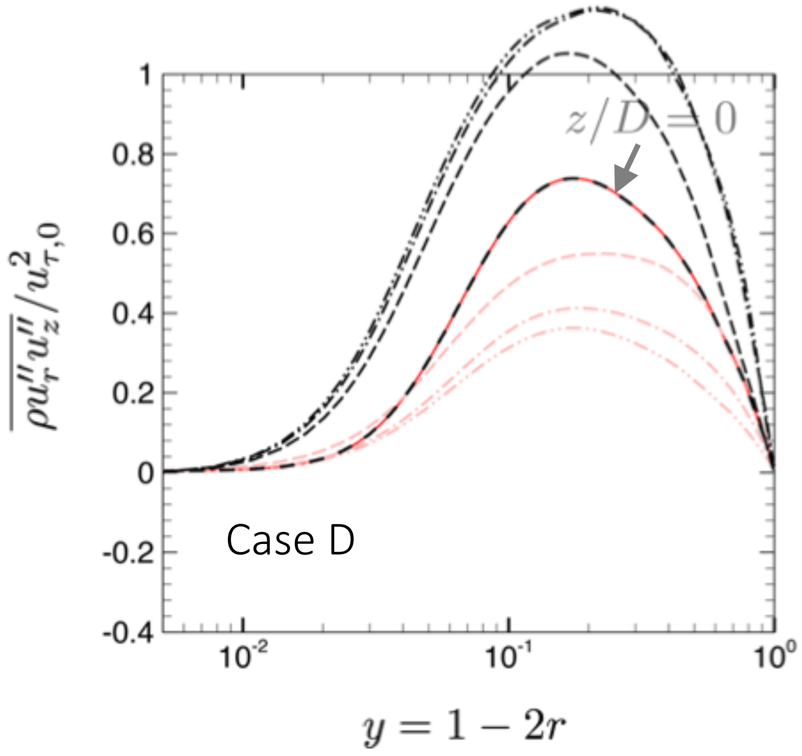


Buoyant Production



Wall temperature distribution from DNS





How do turbulence models perform?

Reynolds/Favre averaged equations

- Momentum equations

$$\frac{\partial \bar{\rho}\tilde{u}_i\tilde{u}_j}{\partial x_j} = -\frac{\partial \bar{p}}{\partial x_i} + \frac{\partial}{\partial x_j}\left[(\bar{\mu}+\mu_t)\, 2\bar{S}_{ij}^c\right] + Ri_{0,z}\bar{\rho}$$

- Enthalpy equation

$$\frac{\partial \bar{\rho}\tilde{h}\tilde{u}_j}{\partial x_j} = \frac{\partial}{\partial x_j}\left[\left(\frac{\bar{\lambda}}{\bar{c}_p} + \frac{\mu_t}{Pr_t}\right)\frac{\partial \tilde{h}}{\partial x_j}\right]$$

- Turbulent kinetic energy equation

$$\frac{\partial \bar{\rho}\tilde{u}_j k}{\partial x_j} = P_k - \bar{\rho}\varepsilon + \frac{\partial}{\partial x_j}\left[\left(\bar{\mu} + \frac{\mu_t}{\sigma_k}\right)\frac{\partial k}{\partial x_j}\right] + B_k$$

Gradient diffusion hypothesis for buoyant production

$$B_k = Gr_{z,0}\beta c_T T_t \left(\frac{2}{3}\bar{\rho}k\delta_{ij} - 2\mu_t S_{ij}^c\right)\frac{\partial T}{\partial x_i}$$

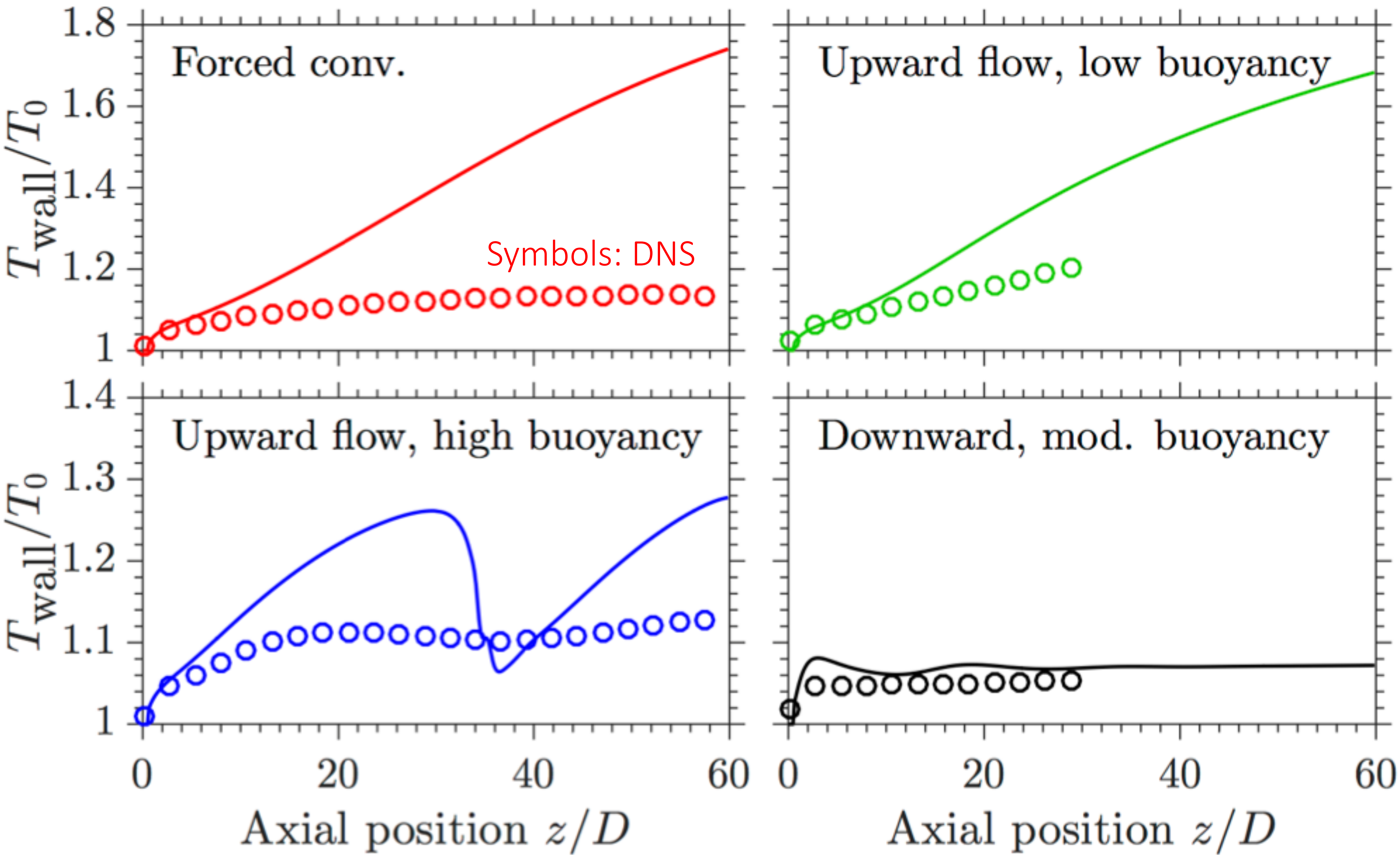
- Other turbulent model supporting equations, for example for V2F model (Durbin 1995)

$$\overline{v'^2}\,, \quad \varepsilon, \quad f$$

- Eddy viscosity

$$\mu_t = C_\mu \bar{\rho}\overline{v'^2}T_t$$

Model results V2F model for supercritical pipe flow

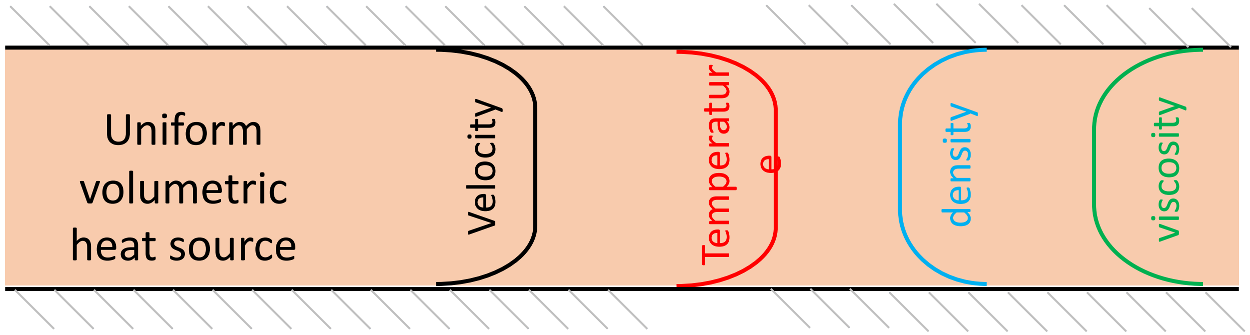


Turbulence model results for supercritical pipe flow

- Model fails to predict even forced convection case
- Other models show similar results (not shown)

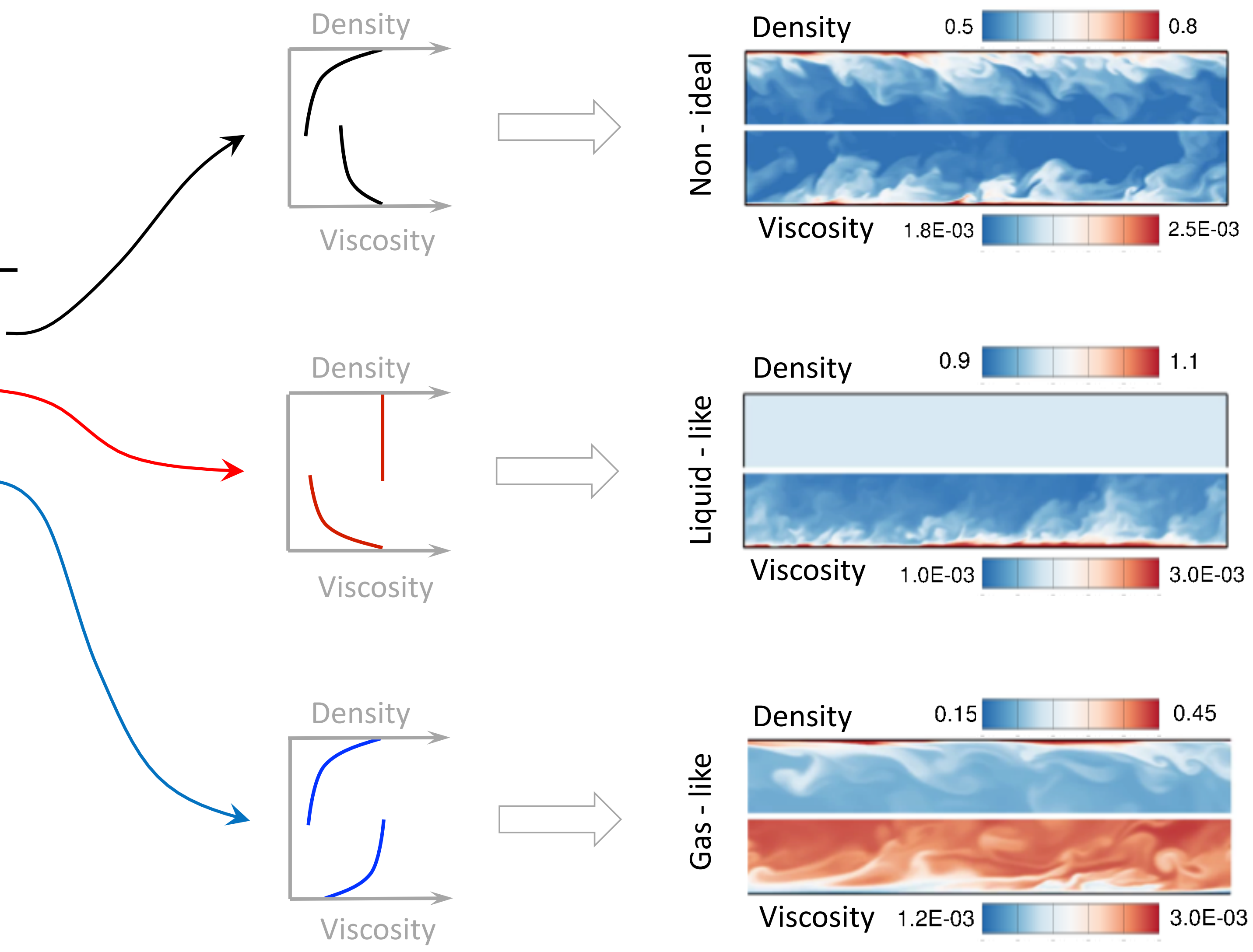
Approach

- First understand how density and viscosity gradients affect turbulence
- Simplified case: volumetrically heated – fully developed – turbulent channel flow



Heated fully developed turbulent channel flows

	Case	Density ρ/ρ_w	Viscosity μ/μ_w
▲	CP395	1	1
—	$CRe_\tau^\star$	$(T/T_w)^{-1}$	$(T/T_w)^{-0.5}$
o	LL	1	$(T/T_w)^{-1}$
····	$SRe_{\tau LL}^\star$	$(T/T_w)^{0.6}$	$(T/T_w)^{-0.75}$
- -	GL	$(T/T_w)^{-1}$	$(T/T_w)^{0.7}$
▫	$SRe_{\tau GL}^\star$	1	$(T/T_w)^{1.2}$



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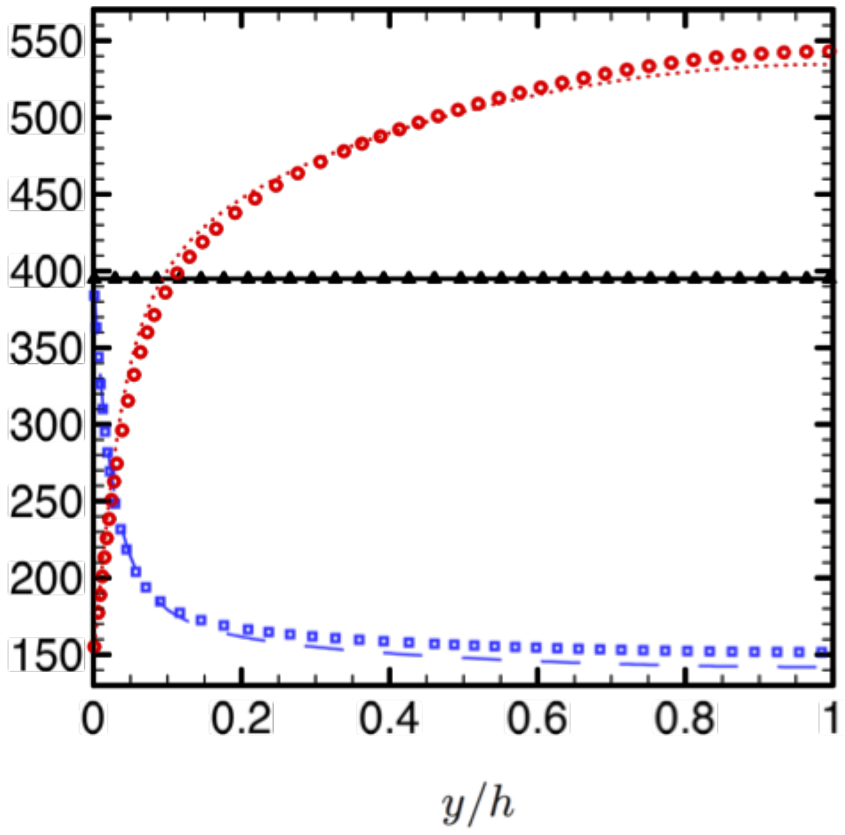
Semi-local Reynolds number

$$Re^\star_\tau \equiv \frac{\sqrt{\langle\rho\rangle/\rho_w}}{\langle\mu\rangle/\mu_w} Re_\tau$$



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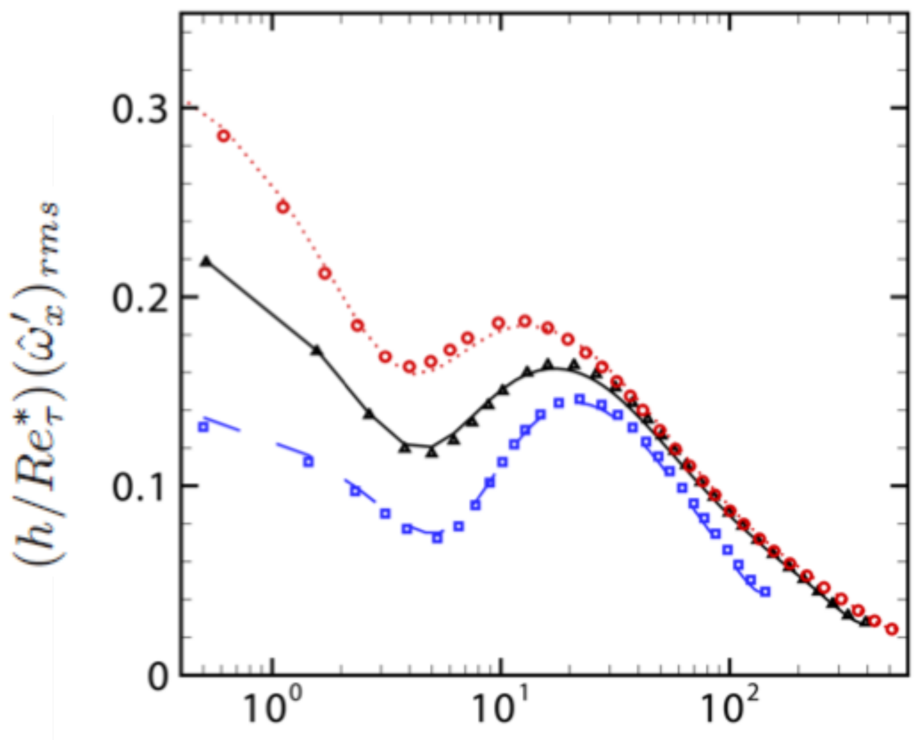
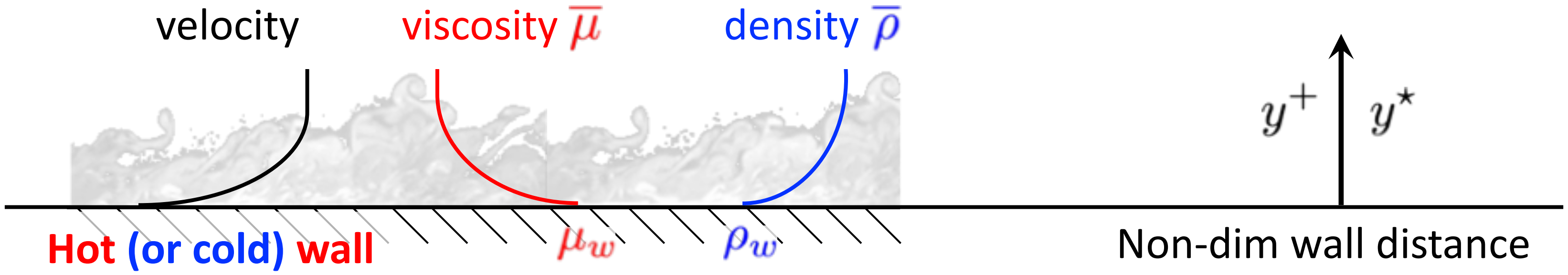
Streamwise vorticity fluctuations



Semi-local wall distance

Semi-local Reynolds number is governing parameter of turbulence statistics!

Semi-local scaling framework



Conventional non-dimensionalization of conservation equations equations

1. Free stream values, i.e. free stream temperature, velocity, density, etc.
2. Wall values, i.e. wall temperature, friction velocity, wall density, etc.

Semi-local non-dimensionalization

- Quantities are normalized by their local values:

$$\widehat{\rho} = \rho/\overline{\rho}, \quad \widehat{\mu} = \mu\,/\overline{\mu}, \quad \widehat{u} = u/u_\tau^\star \qquad \text{with: } u_\tau^\star = \sqrt{\tau_w/\overline{\rho}}$$

Semi-locally scaled conservation equations

additional term

- Continuity:

$$\frac{\partial \hat{\rho}}{\partial \hat{t}} + \frac{\partial \hat{\rho}\hat{u}_i}{\partial \hat{x}_i} + \hat{\rho}\hat{u}_i \underbrace{\frac{1}{2\overline{\rho}}\frac{\partial \overline{\rho}}{\partial \hat{x}_i}}_{C_i} = 0$$

- Momentum:

viscous terms governed by semi-local Reynolds number

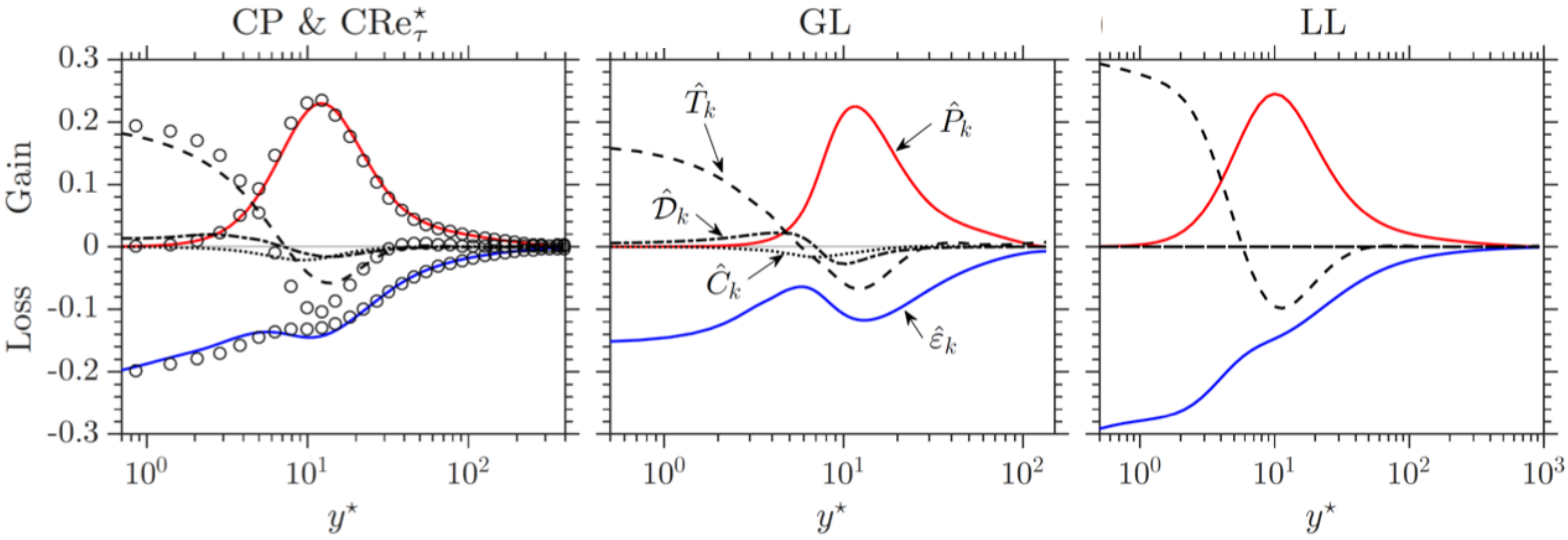
$$\hat{\rho}\frac{\partial \hat{u}_i}{\partial \hat{t}} + \hat{\rho}\hat{u}_j\frac{\partial \hat{u}_i}{\partial \hat{x}_j} - \hat{u}_i\hat{\rho}\hat{u}_j C_j = -\frac{\partial \hat{p}}{\partial \hat{x}_i} + \frac{\partial}{\partial \hat{x}_j}\left[\frac{2\hat{\mu}}{\mathrm{Re}_\tau^\star}\left(\hat{S}_{ij} - \hat{D}_{ij}\right)\right]$$

- Turbulent kinetic energy (fully developed channel):

$$\frac{\partial \overline{\hat{\rho}\hat{k}}}{\partial \hat{t}} + \frac{\partial \overline{\hat{\rho}\hat{k}}\widetilde{\hat{u}}_j}{\partial \hat{x}_j} = -P_k - \overline{\hat{\tau}_{ij}\frac{\partial \hat{u}_i''}{\partial \hat{x}_j}} + \frac{\partial}{\partial \hat{x}_j}\left(\overline{\hat{u}_i''\hat{\tau}_{ij}} - \overline{\hat{\rho}\hat{u}_j''\tfrac{1}{2}\hat{u}_i''\hat{u}_i''}\right)$$
$$+ \hat{u}_j''\overline{\frac{\partial \hat{p}}{\partial \hat{x}_j}} + \left(\overline{\hat{\rho}\hat{k}}\widetilde{\hat{u}}_j + \overline{\hat{\rho}\hat{u}_j''\tfrac{1}{2}\hat{u}_i''\hat{u}_i''}\right) C_j$$

Turbulent budget

$$0 = -P_k - \underbrace{\overline{\hat{\tau}_{ij}\frac{\partial \hat{u}_i''}{\partial \hat{x}_j}}}_{\hat{\varepsilon}_k} + \underbrace{\frac{\partial}{\partial \hat{x}_j}\left(\overline{\hat{u}_i''\hat{\tau}_{ij}} - \overline{\hat{\rho}\hat{u}_j''\tfrac{1}{2}\hat{u}_i''\hat{u}_i''}\right)}_{\hat{T}_k} + \underbrace{\overline{u_j''\frac{\partial \hat{p}}{\partial \hat{x}_j}}}_{\hat{C}_k} + \underbrace{\left(\overline{\hat{\rho}\hat{k}\hat{u}_j} + \overline{\hat{\rho}\hat{u}_j''\tfrac{1}{2}\hat{u}_i''\hat{u}_i''}\right)C_j}_{\hat{D}_k}$$



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Fully developed channel

$$-\frac{\partial}{\partial y}\left[\left(\frac{1}{Re_\tau^\star} + \frac{\hat\mu_t}{\sigma_k}\right)\frac{\partial \hat k}{\partial y}\right] = \hat\mu_t\left(\frac{\partial u^{\mathrm{vD}}}{\partial y}\right)^2 - \hat\varepsilon$$

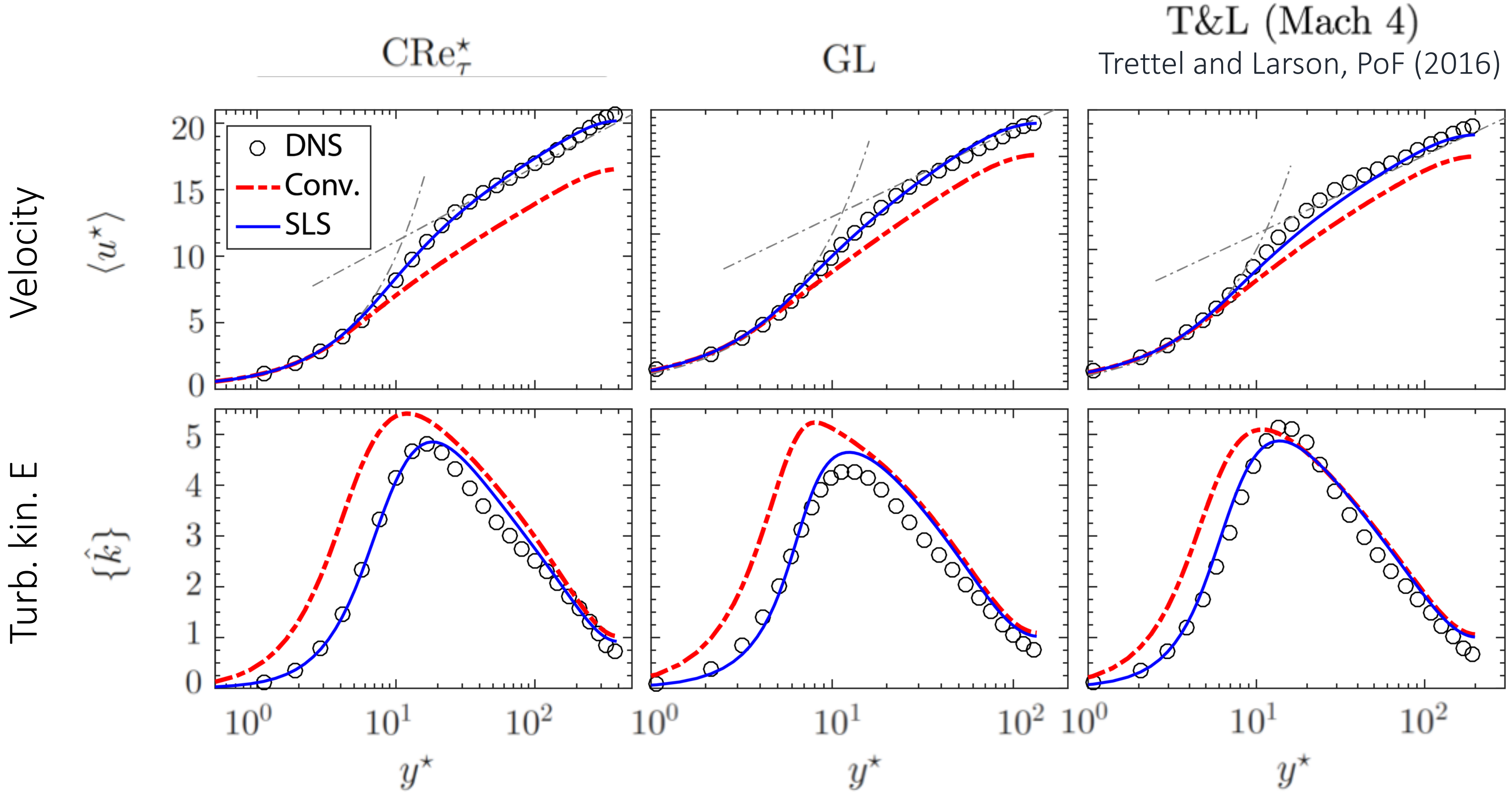
Transforming back to conventional scales

$$\frac{1}{\sqrt{\rho}}\frac{\partial}{\partial x_j}\left[\frac{1}{\sqrt{\rho}}\left(\frac{\mu}{Re_\tau} + \frac{\mu_t}{\sigma_k}\right)\frac{\partial \rho k}{\partial x_j}\right] = \mu_t\left(\frac{\partial u}{\partial y}\right)^2 - \rho\varepsilon$$

Diffusion of TKE acts upon energy per unit volume !

Rodriguez et al., submitted to Int. J. Heat and Fluid Flow, 2018

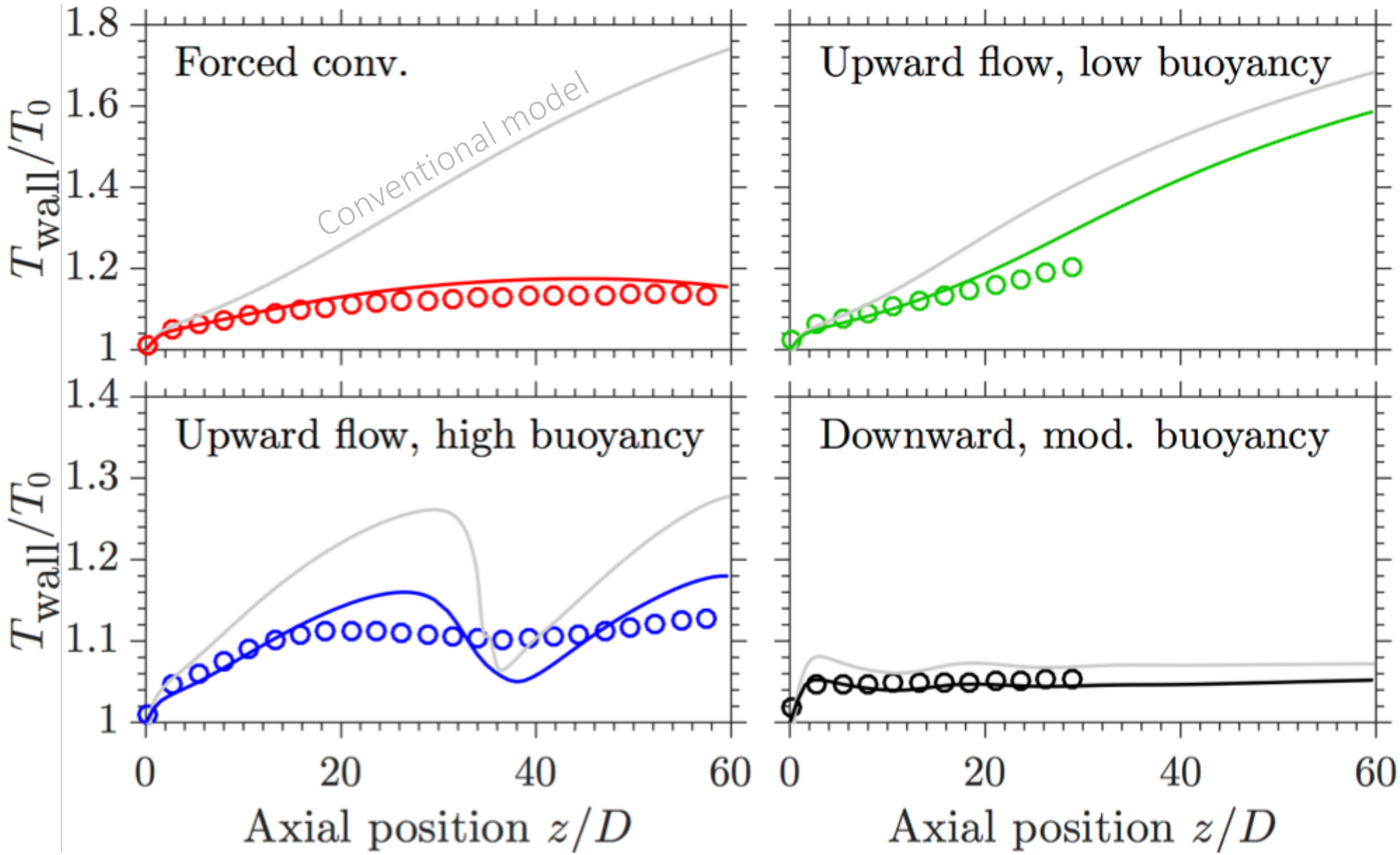
Model results – fully developed turbulent channel



Model: Durbin, 1995

Model results – supercritical pipe flows

Symbols: DNS



Conclusions

- Heat transfer characterization
  - Flow acceleration due to thermal expansion causes decrease in turbulence
  - For upward flows with buoyancy:
    - further decreases turbulence,
    - but after deterioration buoyancy enhances turbulence recovery
  - Downward flows: buoyancy production increases turbulence

- Conventional turbulence models are not capable model heat transfer

- Semi-local framework provides fix when strong gradients in density and viscosity present (no tuning of models and applicable to any turbulence model)

Future directions

- Target modeling buoyancy production term (e.g. flux models)
- Test approach to experimental data @ higher Reynolds number