The Dominant Thermal Resistance Approach for Heat Transfer to Supercritical-Pressure Fluids

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sCO$_2$ Recuperative Recompression Brayton Cycle

'Advanced Design' of the MIT-Study

region of interest
Outline

Introduction

Test Case: Direct Numerical Simulation (Wang, He)

Dominant Thermal Resistance Approach

Results

Conclusions and Outlook
Physical Properties of Supercritical Water

pressure : 23.5 MPa

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Diagram showing properties such as density, thermal conductivity, dynamic viscosity, and enthalpy as functions of temperature ($T$) in degrees Celsius ($^\circ C$). Case 1 is indicated with an arrow pointing to a specific point on the graph.
Why Develop an Approximate Method?

Approach to provide approximate predictions and improved analyses with varying fluid properties

Possibly a useful basis for extending constant property correlations to variable properties

A reasonable, sensible, simple analysis will (may) provide better predictions than empirical correlations for fluids with significant property variation

Improved treatment for wall functions in CFD
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DNS of Supercritical Pipe or Channel Flows (Water or CO$_2$)

DNS : Direct Numerical Simulation
Computational Fluid Dynamics (CFD) without turbulence model, all scales resolved

Rynolds Number Re must below (typically: Re < 6000)

<table>
<thead>
<tr>
<th>authors</th>
<th>case</th>
<th>HT mode</th>
<th>published</th>
<th>year</th>
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<tr>
<td>Bae, Yoo, Choi (Korea)</td>
<td>upward vertical pipe</td>
<td>wall heating</td>
<td>Phys Fluids</td>
<td>2006</td>
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<tr>
<td>Nemati et al. (Delft)</td>
<td>vertical pipe with/no buoyancy</td>
<td>wall heating</td>
<td>IJHMT</td>
<td>2015</td>
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<tr>
<td>Chu and Laurien (Stuttgart)</td>
<td>horizontal pipe</td>
<td>wall heating</td>
<td>J. Supercritical Fluids</td>
<td>2016</td>
</tr>
<tr>
<td>Wang and He (Sheffield, UK)</td>
<td>plane channel with/no buoyancy</td>
<td>constant T at the walls</td>
<td>NURETH-16</td>
<td>2015</td>
</tr>
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<td>Pandey and Laurien (Stuttgart)</td>
<td>vertical pipe</td>
<td>wall heating or cooling</td>
<td>this conference</td>
<td>2018</td>
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</table>
Description of the ‘Wang-DNS’ (no acceleration, no buoyancy)

\[ p = 23.5 \text{ MPa}, \ G = 108.6 \text{ kg/m}^2\text{s}, \ T_c = 367 \ ^\circ\text{C} \]

Geometry case: 1, \( T_h = 367 \ ^\circ\text{C} \)

Turbulence structures visualized by the second-largest eigenvalue of the stress tensor

\[ \text{Re}_b = \frac{u_m}{v_b} 2\delta = 5730 \]

W. Wang & S. He, NURETH-16, 2015
Forced-Convection Correlations for Nu
Dittus Boelter (DB), Gnielinski (VG), Mokry and Pioro (Mok)

\[ \text{Nu} = \frac{q''_w}{(T_w - T_b) k_b} \]

\[ 2 \delta \]

\[ \frac{2 \delta}{(T_w - T_b) k_b} \]

- Wang DNS
- \( \text{Nu}_{DB,b} \)
- \( \text{Nu}_{b,VG} \)
- \( \text{Nu}_{b,Mok} \)

\[ \text{Pr} \]

pseudo-critical temperature 652.5 K at the wall
Forced-Convection Correlations for Nu
Dittus Boelter (DB), Gnielinski (VG), Mokry and Pioro (Mok)

\[ Nu = \frac{q''_w}{(T_w - T_b)} \frac{2\delta}{k_b} \]

\( \text{case 2} \)

\[ \text{pseudo-critical temperature 652.5 K at the wall} \]
Development of direct numerical simulation database for supercritical carbon dioxide

S. Pandey, E. Laurien (Institute of Nuclear Technology and Energy Systems, University of Stuttgart)
X. Chu (Institute of Aerospace Thermodynamics, University of Stuttgart)

Motivation and Aim

- Supercritical carbon dioxide (sCO₂) is a promising working fluid for heat transfer applications.
- Heat transfer peculiarities create a problem in prior prediction, therefore a database of mean statistics is delivered, which is generated by direct numerical simulations.

Numerical and computational details

- Low-Mach Navier-Stokes equations are used instead of the fully compressible N-S equations and finite volume based solver was employed for the DNS.

\[
\frac{\partial p}{\partial t} + \nabla \cdot (p \mathbf{U}) = 0
\]

\[
\frac{\partial p}{\partial t} + \nabla \cdot (p \mathbf{U}) = \rho \mathbf{U} \cdot \nabla \mathbf{U} + \frac{\partial \mathbf{U}}{\partial t} = \mathbf{f} - \nabla p
\]

\[
\frac{\partial \mathbf{U}}{\partial t} + \nabla \cdot \mathbf{U} = 0
\]

- The simulation domain consists of a tube with a total length of 55-65 diameters. Tube diameter ranges from 2-10 mm.

Results

- During heating, a local peak in temperature can be seen in upward flow, which is a characteristic of heat transfer deterioration.

\[
\text{C}_{10} = \text{constant}
\]

- Total 47 cases were simulated by means of DNS. This includes heating and cooling in the vertical orientation of tube. For heating, we conducted DNS only for upward flow with combinations of inlet temperature, pressure, diameter, and heat flux. For cooling, all three possibilities (upward, downward and forced) were simulated for 2 mm diameter.

- During cooling, downward flow case suffers from the heat transfer deterioration. The ejection and sweep events reduced in the downward flow case which attenuate the turbulence, thereby, the heat transfer.

- Streak stretching in the downward flow (in cooling) leads to impaired heat transfer.

- This database can be used in benchmarking of turbulence models, empirical fitting of the coefficient in correlations, and training of machine learning algorithm.

The 6th International Supercritical CO₂ Power Cycles • March 26-29, 2018 • Pittsburgh, PA, USA
Importance of the Turbulent Core
DNS data from Bae, Yoo and Choi, Phys. Fluids 2005

\[
\frac{T_w - T(y)}{T_w - T_b} \approx 5\% 
\]

\[ r / R \]

DNS
Bae et al. (2005)
no buoyancy
\( p = 8 \, \text{MPa} \)
\( R_w = 1 \, \text{mm} \)
\( R_{\theta_b} = 6560 \)
\( T_0 = 28\, ^{\circ}\text{C} \)
\( x/R_w = 58 \)
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Dominant Thermal Resistance Approach

assumptions

Steady state,

Quasi-established velocity and temperature profiles

Constant shear layer and heat flux layer approximations

Negligible buoyancy, negligible acceleration

Turbulent core - high turbulence,

\[ T_b \approx T_{\text{centerline}} \approx T_{\text{laminar sublayer}} = T_{\text{conducting sublayer}} \]
Wang-DNS Mean Temperature Profile
Case 1

region of interest

left wall

\[ \frac{T}{T_{ref}} \]

cooled side

heated side

right wall

wall distance

conducting sub-layer

\( y_{cs} \)
Wang-DNS Mean Velocity Profile
Case 1

region of interest

\[ \bar{u} \]

\[ 2\delta \]

\[ \delta \]

wall distance

heated side

cooled side

viscous sub-layer

\[ y_{vs} \]
Integration of the Thermal Energy Equation
in the laminar, conducting sub-layer: the region of dominant thermal resistance

\[ 0 = \rho c_p U \frac{\partial T}{\partial x} \approx \frac{\partial q''}{\partial y} \Rightarrow q''(y) \approx \text{const.} \approx q_w \]

Near the wall we have Fourier's law:

\[ q''(y) \approx -k(T) \frac{\partial T}{\partial y} \quad \text{Integrate:} \quad q''(y) \int_0^y d\ y \approx -\int_{T_w}^{T(y)} k(T) \ d T \]

Define

\[ \omega(T) = -\int_{T_{\text{ref}}}^T k(T) \ d T \quad \text{a property} \]

then

\[ q_w' \ y = -\left[ \omega(T) - \omega(T_w) \right] \]

At \( y = y_{cs} \)

\[ q_w'' = -\frac{\omega(T_b) - \omega(T_w)}{y_{cs}} \quad \text{May be a good approximation} \]

If one has a good estimate of \( y_{cs} \)
How to get good Estimates for $y_{cs}$ using the universal wall units

Prandtl [1910] approach $y_{cs}^+ \approx y_{vs}^+$

Two-layer approximation $y_{vs}^+ \approx 11.6$

where $y^+ = y \left(\frac{\tau_w}{\rho}\right)^{1/2}/\nu$ or

$y^+ = \left(\frac{y}{D_h}\right) \text{Re}_{Dh} \left(C_f/2\right)^{1/2}$

DO WE HAVE A GOOD ESTIMATE OF $C_f$?
Forced-Convection Correlations for wall friction $c_f$
used in Gnielinski (VG), Drew, Koo, McAdams (DKM), M.F. Taylor (MFT), Blasius (Blas)

$$c_f = \frac{\tau_w}{0.5 \rho_w u_m^2}$$

case 1

- Wang DNS
Forced-Convection Correlations for Nu
used in Gnielinski (VG), Drew, Koo, McAdams (DKM), M.F. Taylor (MFT), Blasius (Blas)

\[ c_f = \frac{\tau_w}{0.5 \rho_w u_m^2} \]

Case 2

- Wang DNS

\[ C_{f,b, VG} \]

\[ C_{f,DKM,b} \]

\[ C_{f,Blas,b} \]

\[ C_{f,MFT,b} \]
Integration of the Momentum Equation
in the laminar, conducting sub-layer: the region of dominant flow resistance

\[ \tau(y) \approx \mu \frac{\partial U}{\partial y} \Rightarrow \tau(y) \approx \text{const.} \approx \tau_w \]

Near the wall we have Newton's law and Fourier's law:

\[ \tau_w \, dy = \mu(T) \, dU \quad \text{and} \quad q'' \, dy = -k(T) \, dT \]

\[ dU = \frac{\tau_w \, dy}{\mu(T)} \quad \text{and} \quad dy = -\frac{k(T) \, dT}{q''} \Rightarrow dU = -\frac{\tau_w \, k(T) \, dT}{\mu(T) \, q''} \]

Define

\[ \Phi(T) = -\int_{T_{\text{ref}}}^{T} \frac{k(T)}{\mu(T)} \, dT \quad \text{a property} \]

Integrate

\[ \int_{0}^{U_b} dU \approx -\tau_w \int_{T_w}^{T_b} \frac{k(T)}{q'' \mu(T)} \, dT \Rightarrow U_b = -\frac{\tau_w}{q''} \left[ \Phi(T_b) - \Phi(T_w) \right] \]

Solve for

\[ \tau_w = -\frac{U_b \, q''}{\Phi(T_b) - \Phi(T_w)} \]
Integration of the Momentum Equation (contd.)

in the laminar, conducting sub-layer: the region of dominant flow resistance

Substitute into

\[ q_w'' y_{cs} = - \left[ \omega(T_{cs}) - \omega(T_w) \right] \]

with

\[ y_{cs} = \frac{y_{cs}^+ \nu}{\sqrt{\tau_w/\rho}} \]

e.g. into

\[ q_w'' \frac{y_{cs}^+ \nu}{\sqrt{\tau_w/\rho}} = - \left[ \omega(T_{cs}) - \omega(T_w) \right] \]

to give

\[ q_w'' \frac{y_{cs}^+ \nu}{\sqrt{\frac{U_b q_w''}{\Phi(T_w) - \Phi(T_b)} / \rho}} = - \left[ \omega(T_{cs}) - \omega(T_w) \right] \]

and solve for

\[ q_w'' = \frac{U_b}{\rho (y_{cs}^+ \nu)^2} \frac{\left[ \omega(T_b) - \omega(T_w) \right]^2}{\left[ \Phi(T_w) - \Phi(T_b) \right]} \]
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Result
Heat Transfer

$$Nu = \frac{q''_w}{(T_w - T_b)^2} \frac{2\delta}{k_b}$$

case 1

$$y^+_v = y^+_c = 11.6$$

$$y^+ = \frac{y}{\nu_w \sqrt{\frac{\tau_w}{\rho_w}}}$$ upper index + in „wall units“
Result
Heat Transfer

\[ Nu = \frac{q''_w}{(T_w - T_b)} \frac{2\delta}{k_b} \]

*case 2*

\[ \text{Nu} \]

\[ T_w / K \]

- Wang DNS
- Present
Friction

\[ c_f = \frac{\tau_w}{0.5 \rho_w u_m^2} \]

**Result**

- **case 3**

![Graph showing friction coefficient \(c_f\) vs. \(T_w/K\).](image)

- **Wang DNS**
- **Present**
- **C \_f,\_b,\_VG**
- **C \_f,\_DKM,b**
- **C \_f,\_Blas,b**
- **C \_f,\_Bud,b**
Result

Heat Transfer

\[ Nu = \frac{q''_w}{(T_w - T_b) k_b} \frac{2\delta}{w_k} \]

Case 3

\[ T_h = 655 \text{ K} \]

\[ \begin{align*}
N_u & \text{ present} \\
\end{align*} \]
Concluding Remarks

Demonstrated a closed-form, approximate, coupled analysis for Nu for ScPF (with negligible buoyancy and acceleration)

Some reasonable agreement with DNS of Wang+He

Nu is sensitive to choice of $y_{v_s}^+$

Useful approach to provide approximate predictions and improved analyses

Improved treatment for wall functions in CFD

Can provide a first estimate for interative processes in "more sophisticated" analyses
Outlook

Extend to significant buoyancy and acceleration

Revise analysis to treat differing $y_{cs}^+$ and $y_{vs}^+$

Add thermal resistance for turbulent core?
DNS data: 80 bar, CO$_2$, D = 2 mm

**upward**

- $q_W = 61$ kW/m$^2$K

**downward**

- $q_W = 61$ kW/m$^2$K
Backup Slides
State-of-the-Art Correlations for Narrow Channels (2 mm)

$G = 60 \text{ kg/m}^2\text{s}, q_w = -30 \text{ kW/m}^2$ (cooled wall)

**Forced convection**

$$Nu_0 = Nu_0 \left( \frac{\rho_w}{\rho_b} \right)^{n_1} \left( \frac{c_{pw}}{c_{pb}} \right)^{n_2} \ldots$$

- **index 0**: constant properties
- **w**: wall
- **b**: bulk

**Mixed convection (w/ gravity influence)**

$$Nu = Nu_0 \left[ 1 \pm C_1 \times \left( \frac{Gr_m}{Re^{2.7}} \right)^{n_1} \right] \ldots$$

- **Nu**: Nusselt number
- **Gr**: Grashof number
- **Re**: Reynolds number
- + upward, - downward flow

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Graphs showing temperature $T_b$ vs. bulk fluid enthalpy $[\text{kJ/kg}]$ for different models:

- Bulk-Temperatur
- Dittus-Boelter
- Gnielinski
- Krasnoshchekov
- Mokry u. Pioro

Almost the same result for upward and downward flow.
DNS-Results of a Heated PipeFlow at Re = 5400

upwards: thermally stable
buoyancy first damps later induces turbulence (recovery)

downwards: thermally unstable
buoyancy induces turbulence

Visualization of turbulence structures using the $\lambda_2$ criterium
Terminology
of Single-Pipe or Channel Experiments at Super-Critical Pressure

- pseudo-critical point \((\rho_{pc}, T_{pc})\)
- pseudo-critical line
- critical point
- saturation line
- bulk temperature \((T_b)\) of a heated/cooled pipe/channel
- heating
- cooling

\(T \, [^\circ C]\)

\(p \, [\text{bar}]\)

\(\rho = 100 \, \text{kg/m}^3\)
sCO$_2$ Recuperative Recompression Brayton Cycle

'Advanced Design' of the MIT-Study: Net Efficiency ~47%

Diagram showing the cycle with key components:
- Input HX
- High Temp Recuperator
- Alternator
- Low Temp Recuperator
- Re-Compressor
- Main Compressor
- Turbine

Table of pressures and temperatures:

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<thead>
<tr>
<th>p MPa</th>
<th>T °C</th>
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<td>1</td>
<td>7,7</td>
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<tr>
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<td>7</td>
<td>7,7</td>
</tr>
<tr>
<td>8</td>
<td>7,7</td>
</tr>
</tbody>
</table>

Critical point and region of interest highlighted.
The temperature in wall units

\[ T^+ (y) = \frac{\rho c_p u_\tau}{q_w''} \left( T_w - T(y) \right) = \rho c_p u_\tau R \]

Can be considered a non-dimensional thermal resistance. Expand to

\[ T^+ (y) = \rho c_p u_\tau \left( R_{lam} + R_{turb} \right) \]

And compare to the Kays and Crawford relation

\[ T^+ (R) = Pr y_{lam}^+ + \frac{Pr_t}{\kappa} \left( \ln R^+ - \ln y_{lam} \right) \approx T_b^+ \]

The total resistance is the sum of two individual resistances for the two layers.
Water, $y^+\{T\}_w$

$23.5 \text{ MPa, } T_b = 647.4 \text{ } ^\circ\text{C}$

$Re_{Dh,b} = 5210$

- $C_{f,b,2L}$
- $C_{f,b,VG}$
- $C_{f,Bud,b}$
- $C_{f,DKM,b}$
- $C_{f,Blas,b}$

Wang DNS
Wang DNS

Water, $y^+\{T\}_w$

$23.5$ MPa, $T_b = 652.4$ C

$Re_{Dh,b} = 1180$

$C_{f,b,2L}$

$C_{f,b,VG}$

$C_{f,DKM,b}$

$C_{f,Blas,b}$

$C_{f,Bud,b}$