Effects of thermal boundary condition on turbulent statistics in flows with a supercritical fluid

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Motivation

Several numerical studies on heat transfer to supercritical fluids

- Effect of buoyancy, heat flux/mass flux ratio, etc.
- Yoo, Annual Review Fluid Mechanics, 2013

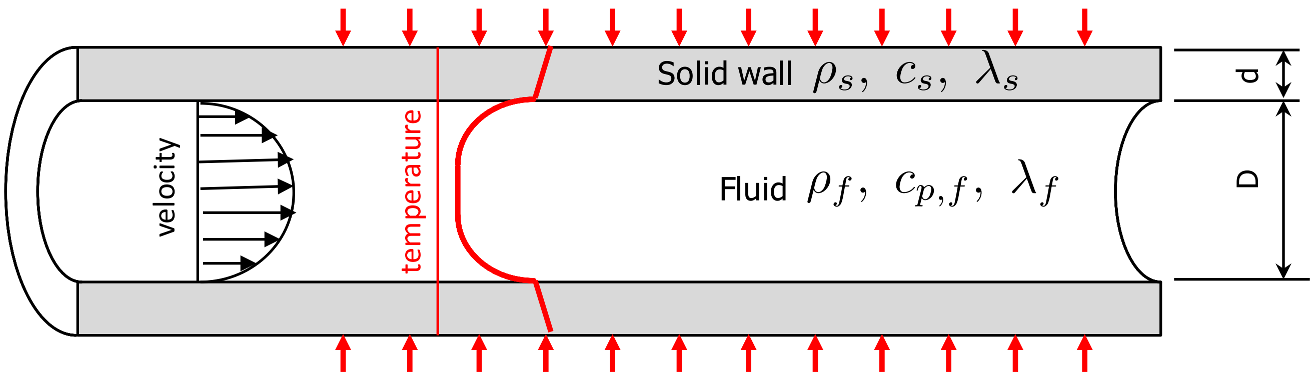
Most of the numerical studies assume isoflux boundary conditions

- Isoflux BC allows temperature fluctuations at the wall
- Isothermal BC does not allow temperature to fluctuate at the wall

If fluid's Prandtl number > 1, temperature fluctuations do not affect heat transfer (Kasagi, 1989; Li et al., 2009).

Does this also hold for flows with strong property gradients even if Pr > 1?

Effect of fluid/wall properties on temperature fluctuations

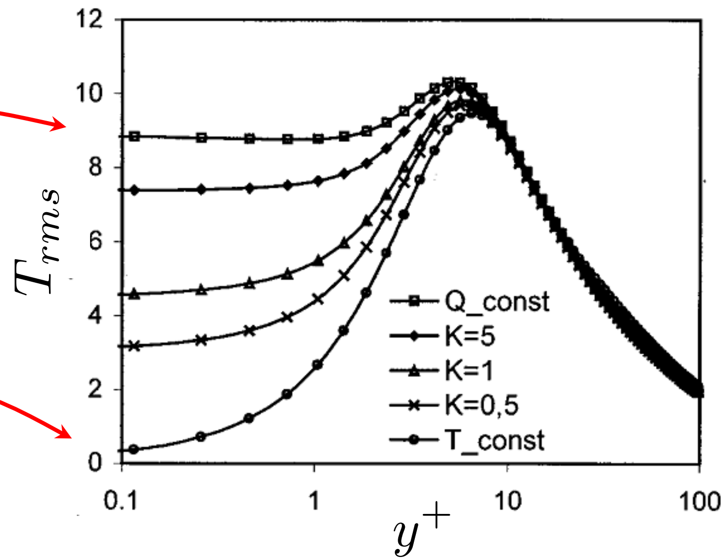


Thermal effusivity ratio:

$$K = \sqrt{\frac{\rho_f c_{p,f} \lambda_f}{\rho_s c_s \lambda_s}} \to \infty : \text{ isoflux BC}$$

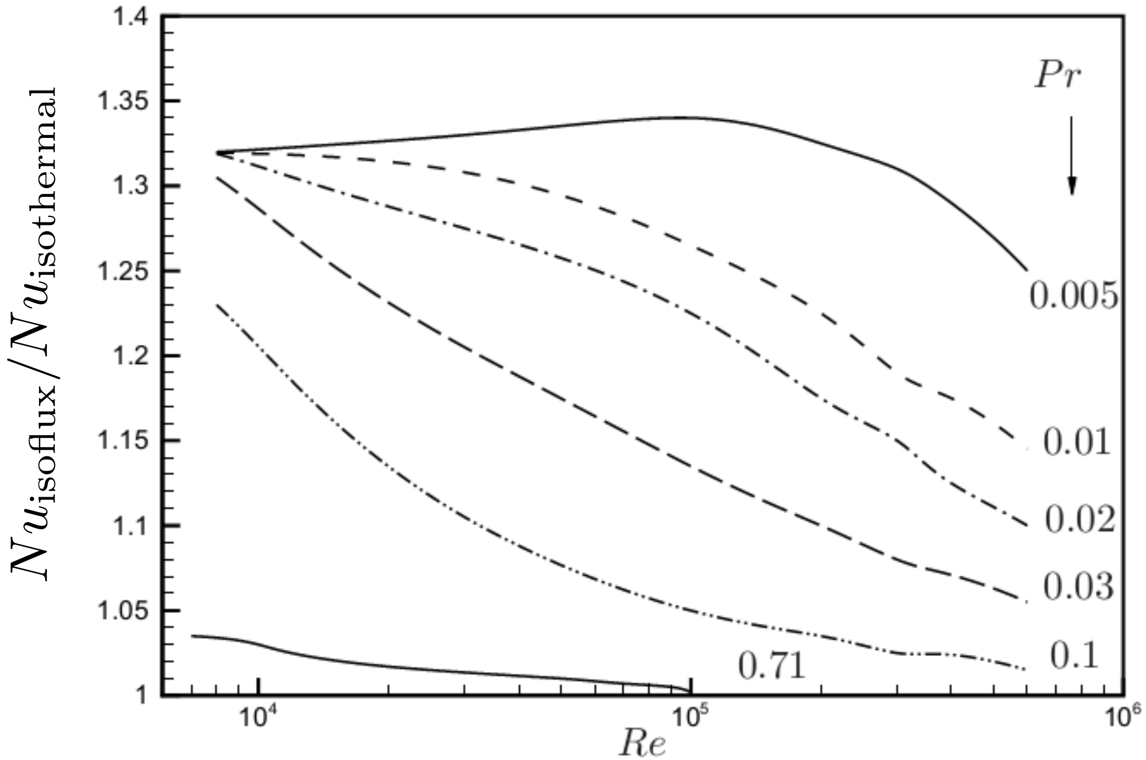
$$K = \sqrt{\frac{\rho_f c_{p,f} \lambda_f}{\rho_s c_s \lambda_s}} \to 0 : \text{ isothermal BC}$$

From Tiselj et al. 2001, JHT



Effect of Prandtl number

- Ratio of Nusselt number for isoflux to isothermal boundary conditions



Sleicher, 1955; Kasagi et al., 1989

Thermal effusivity ratio and Prandtl number examples

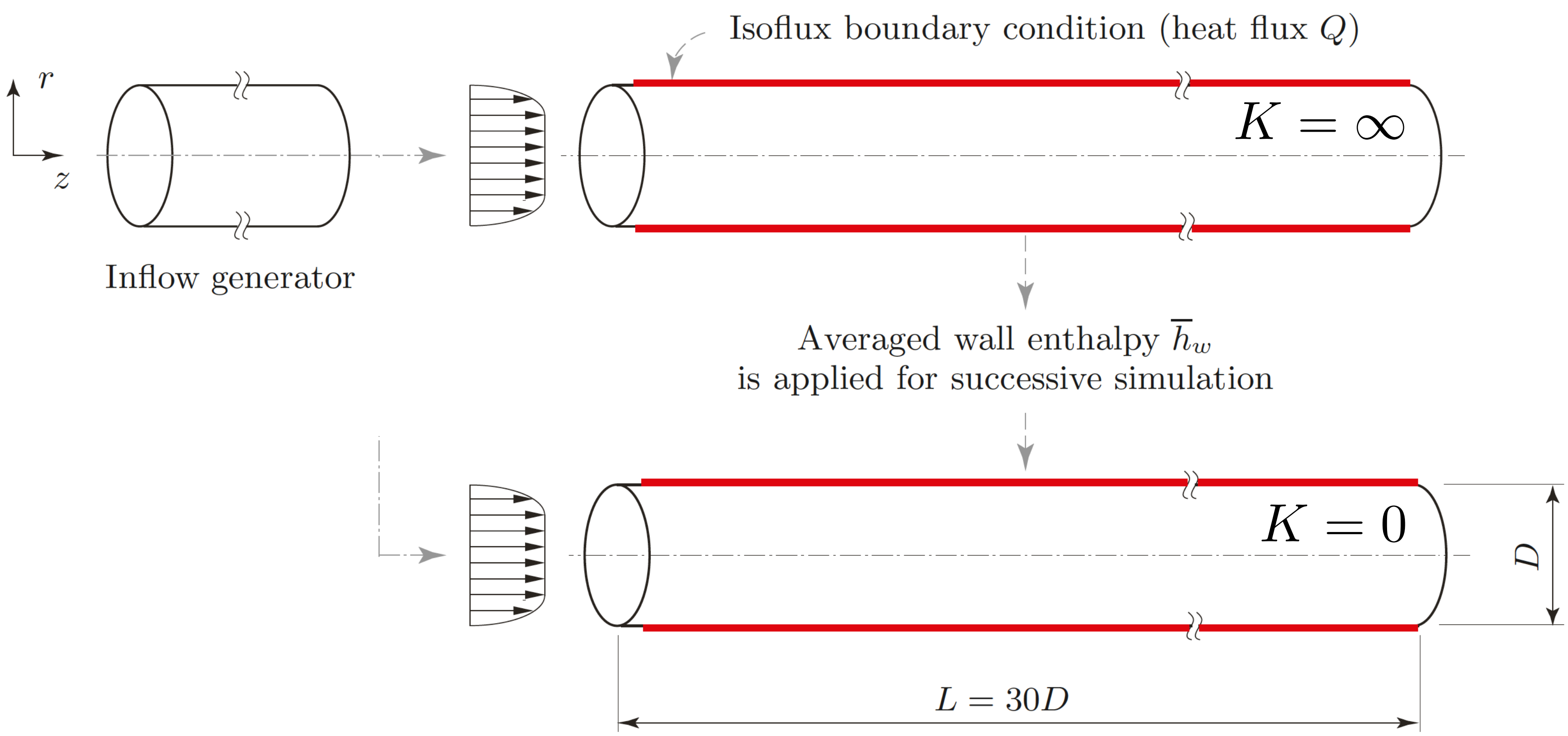
Prandtl number	Air * 0.708	Water * 6.78	$scCO_2$ (80bar) up to 14
Aluminum	0.00025	0.071	
Nickel based alloy	0.00073	0.207	~0.25
Copper	0.00015	0.044	
Glass	0.00419	1.190	
Plexiglas	0.00942	2.680	

* based on Kasagi et al., Journal of heat transfer, 1989

Investigate influence of thermal effusivity ratio on heat transfer to $scCO_2$

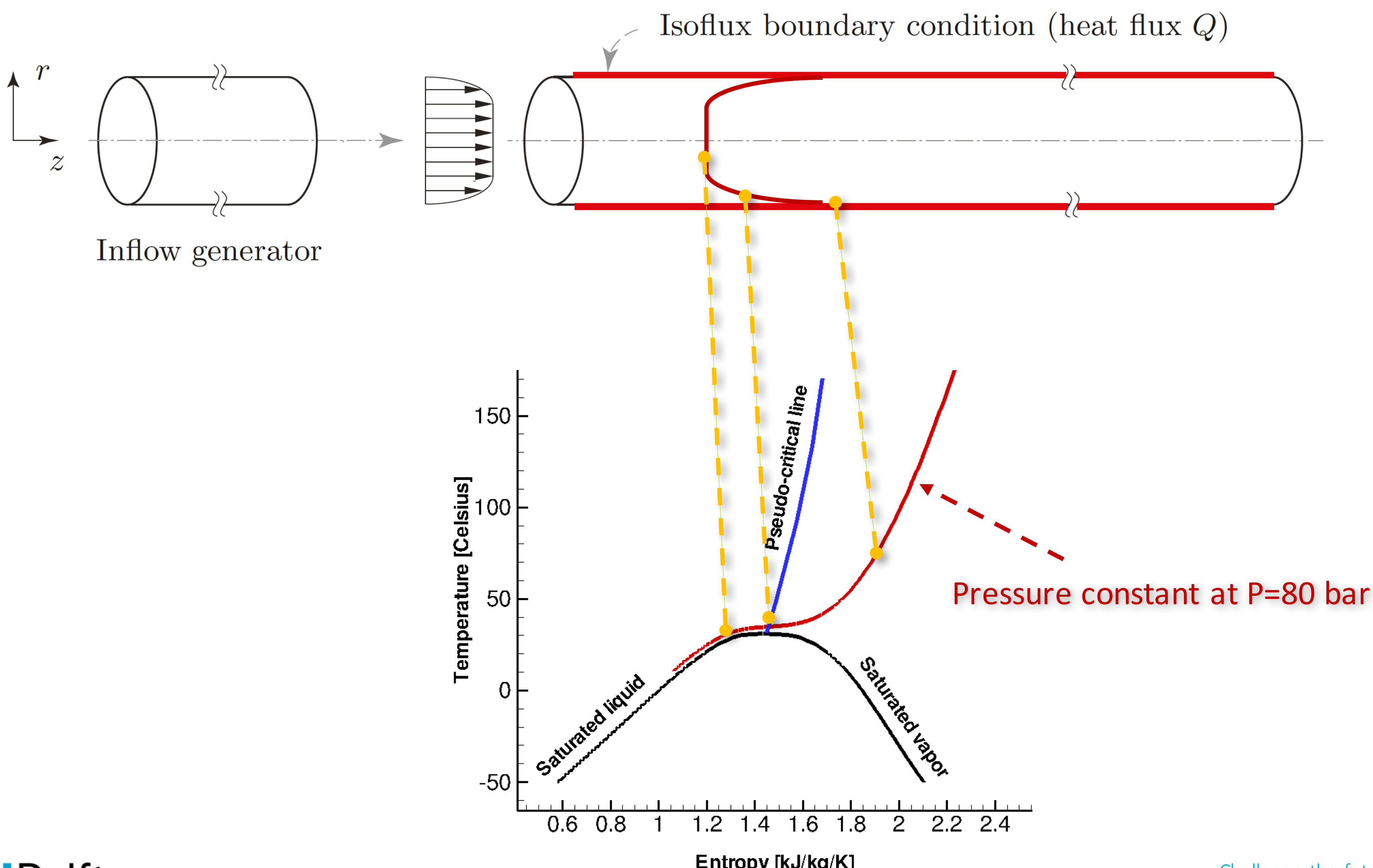
- Allow wall temperature fluctuations: $K = \infty$
- Do not allow temperature fluctuations: $K = 0$

Simulation setup

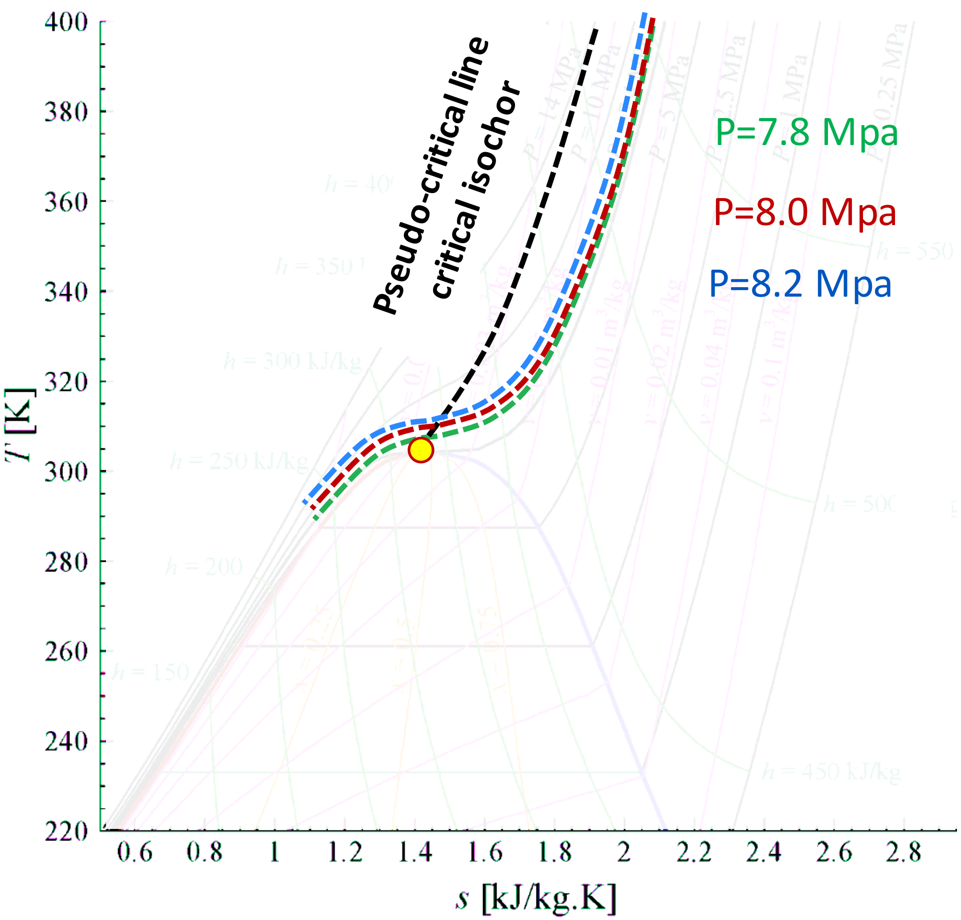


This setup ensures the same thermodynamic condition at the wall!

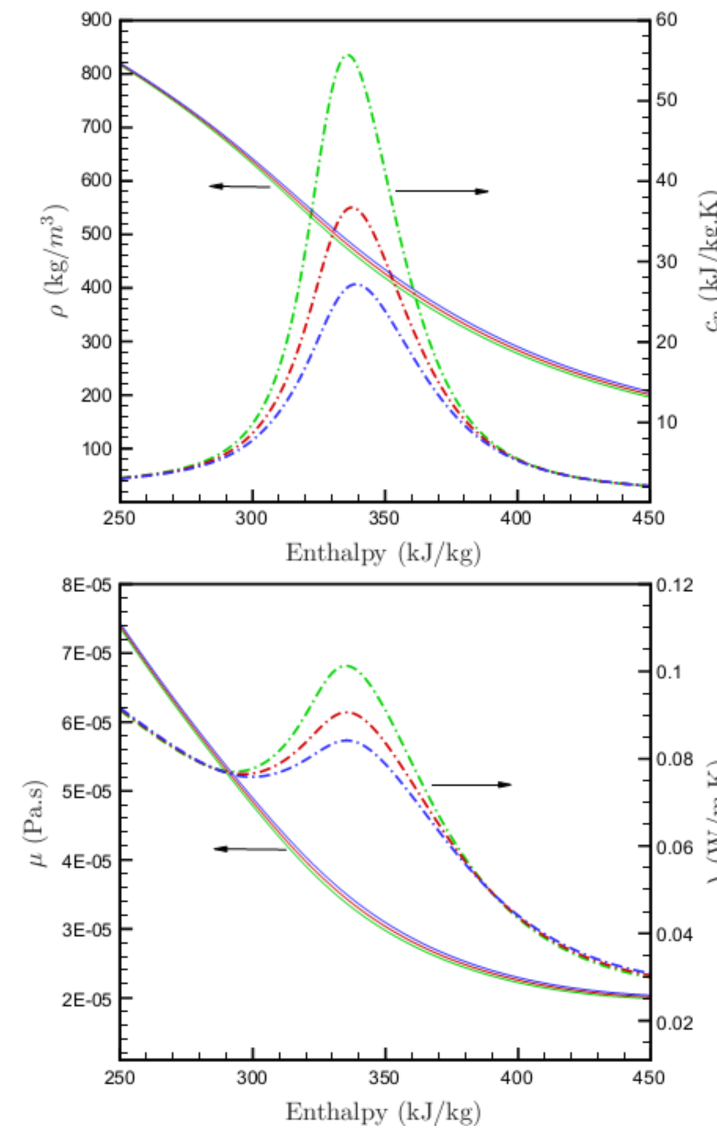
Simulation setup



Properties of supercritical fluids



T-s Diagram including the critical point for CO_2



Governing equations

Low-Mach number approximation of Navier-Stokes equations:

$$\frac{\partial \rho}{\partial t} + \frac{\partial \rho u_i}{\partial x_i} = 0$$

$$\frac{\partial \rho u_i}{\partial t} + \frac{\partial \rho u_i u_j}{\partial x_j} = -\frac{\partial p}{\partial x_i} + \frac{1}{Re_{\tau 0}} \frac{\partial \tau_{ij}}{\partial x_j} \qquad \tau_{ij} = \mu S_{ij} = \mu\left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i} - \frac{2}{3}\frac{\partial u_k}{\partial x_k}\delta_{ij}\right)$$

$$\frac{\partial \rho h}{\partial t} + \frac{\partial \rho u_i h}{\partial x_i} = -\frac{1}{Re_{\tau 0} Pr_0} \frac{\partial q_i}{\partial x_i} \qquad q_i = -\lambda \frac{\partial T}{\partial x_i} = -\frac{\lambda}{c_p}\frac{\partial h}{\partial x_i} = -\alpha \frac{\partial h}{\partial x_i}$$

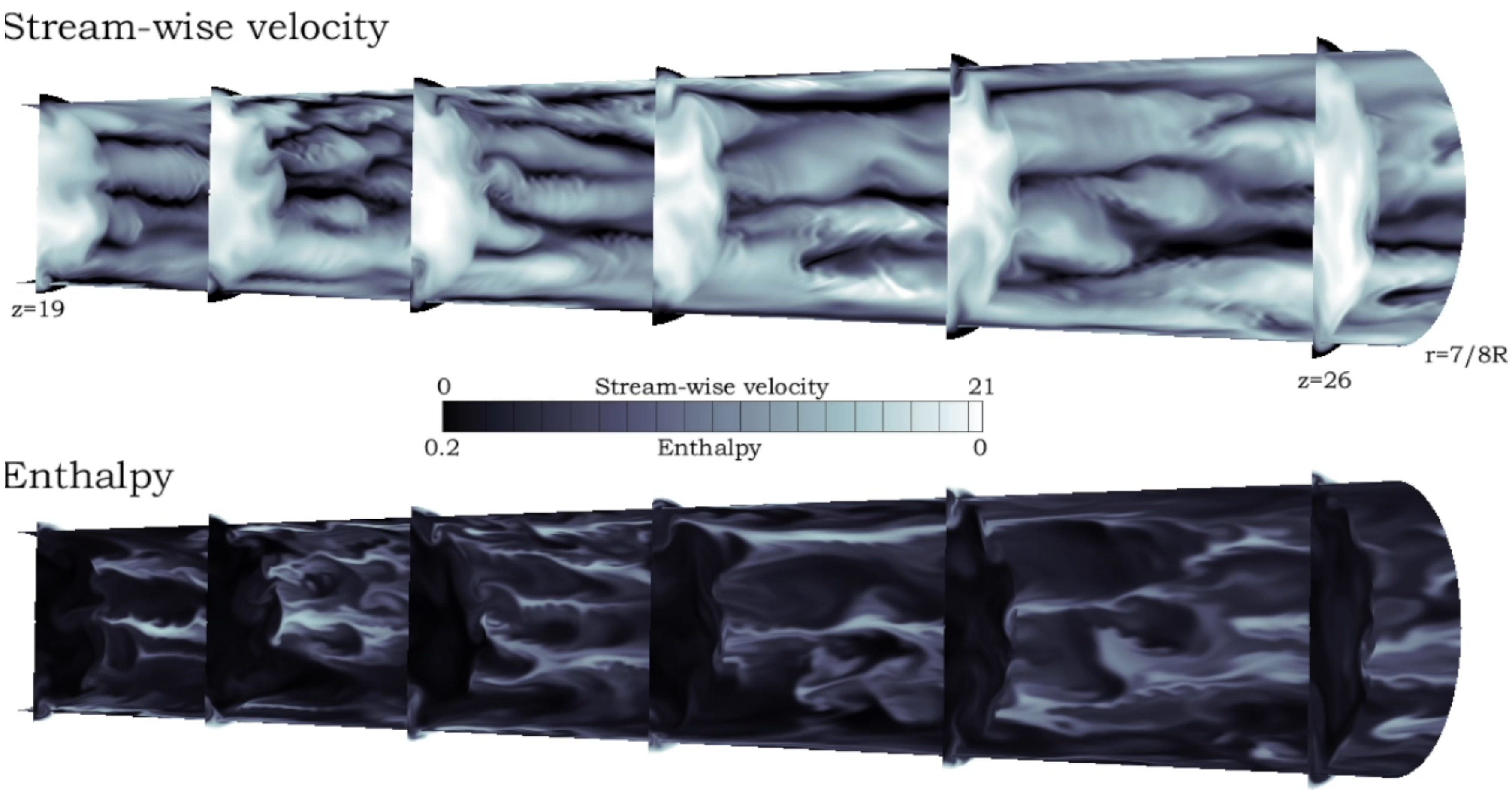
with:

$$Re_{\tau 0} = \frac{\rho_0^* u_{\tau 0}^* D^*}{\mu_0^*} = 360 \qquad Pr_0 = \frac{\mu_0^* c_{p0}^*}{\lambda_0^*} = 3.2 \qquad Q = \frac{q_w^* D^*}{\lambda_0^* T_0^*} = Re_{\tau 0} Prq = 2.4$$

Numerical scheme

- **Spatial discretization**: 2^{nd} order central difference on staggered mesh
- **Temporal discretization**: 2^{nd} Adams-Bashforth and Adams-Moulton
- Koren limiter for advection part of energy equation
- Diffusion part in circumferential direction treated implicitly
- Mesh resolution:
  - Mesh points 128 x 288 x 1728
  - Radial $0.55\ (wall) < \Delta r^{+} < 4.3\ (center)$
  - Circumferential $R\Delta\theta^{+} = 3.93$
  - Axial $\Delta z^{+} = 6.25$
- **Thermophysical properties (CO_2 at P=8 Mpa)** are interpolated from table

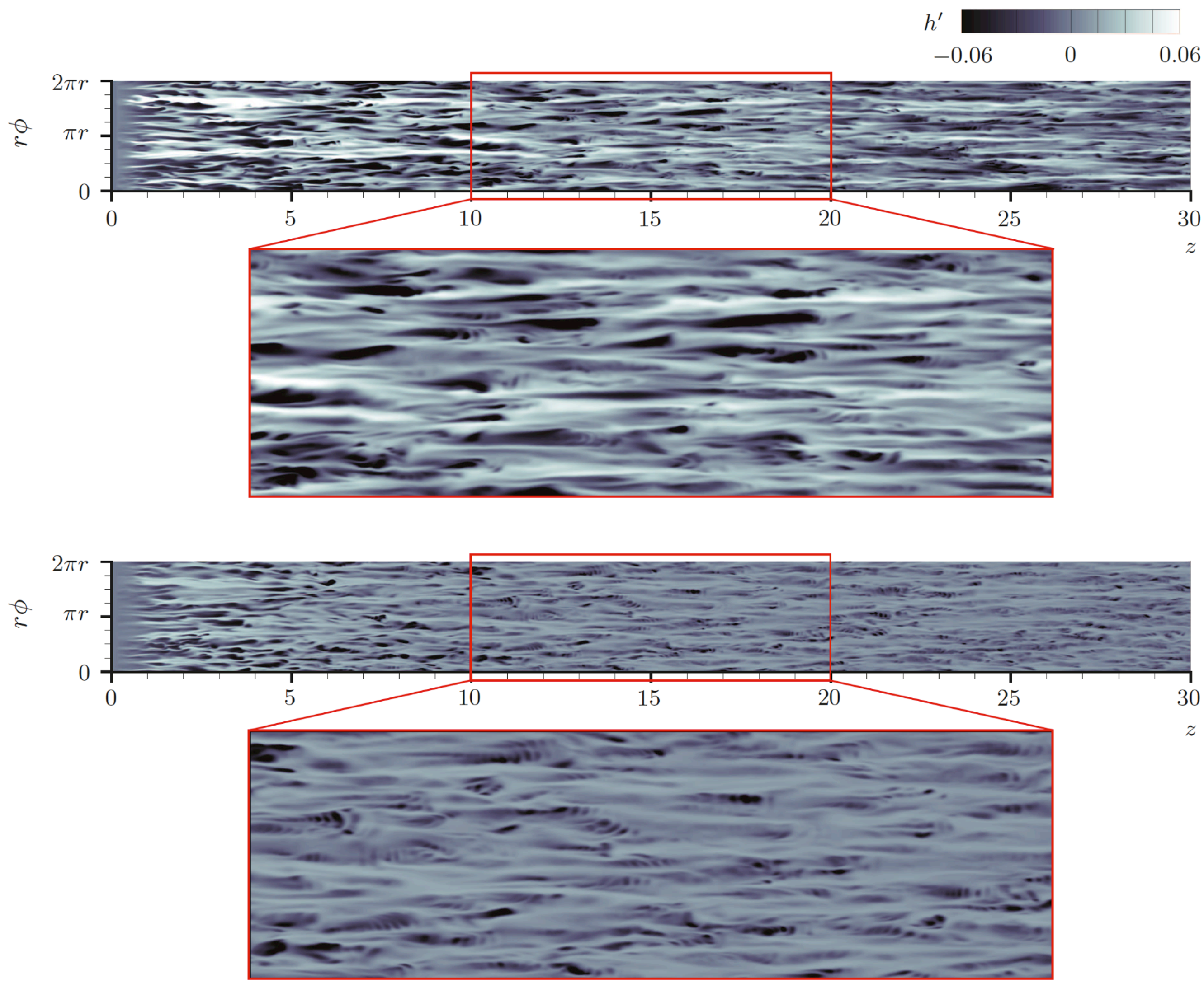
Instantaneous flow field, isoflux simulation



Instantaneous enthalpy fluctuations

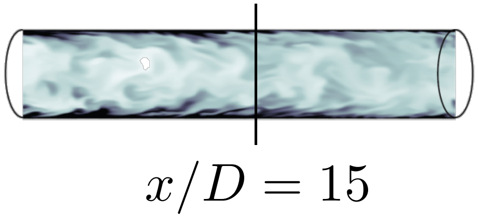
Iso-flux

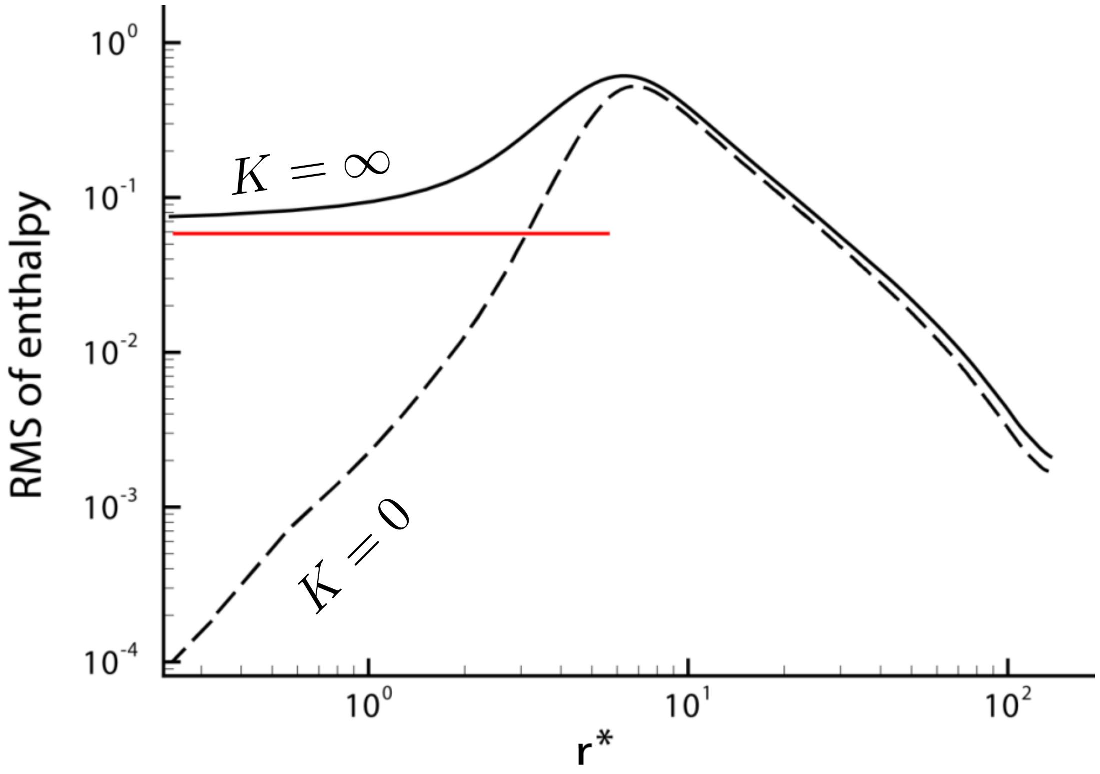
Iso-thermal



$y^+ = 4.7$ (based on inlet condition)

Enthalpy rms profiles

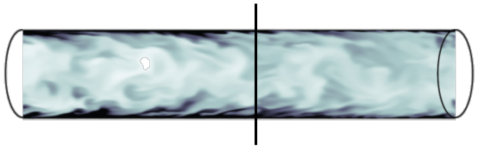




Radial heat fluxes

Total radial heat flux:

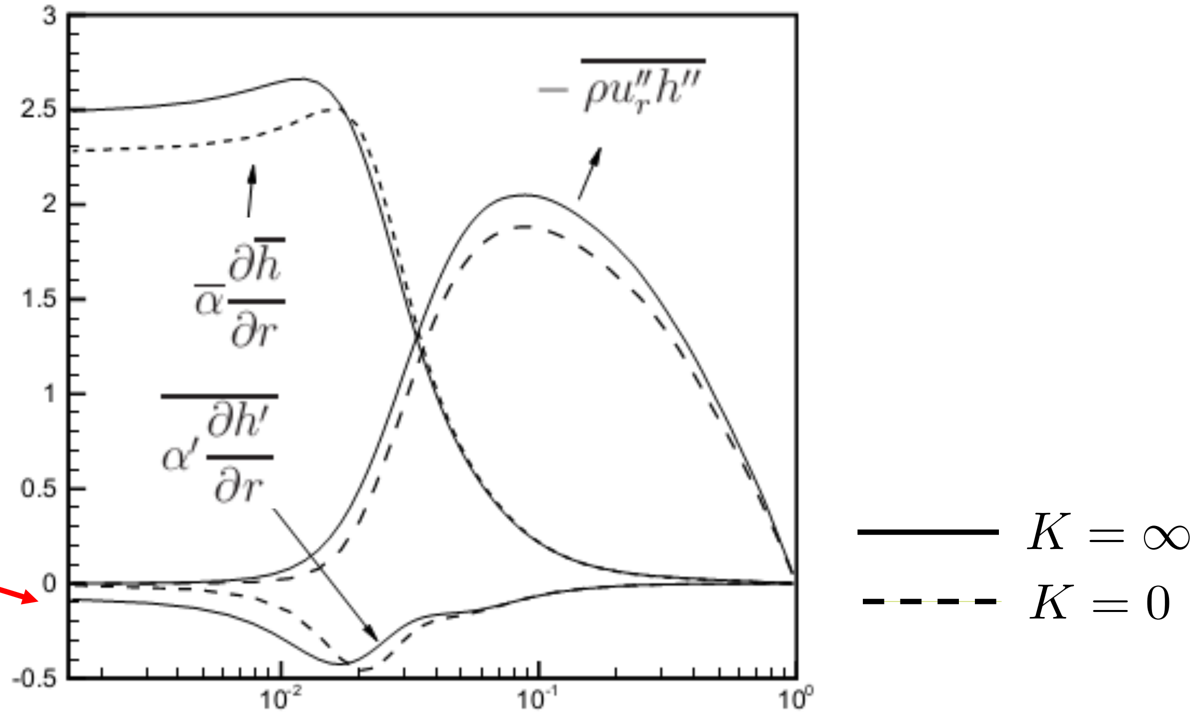
$$q_{r,tot} = \overline{\alpha}\frac{\partial \overline{h}}{\partial r} + \overline{\alpha' \frac{\partial h'}{\partial r}} - \overline{\rho u_r'' h''}$$



$x/D = 15$

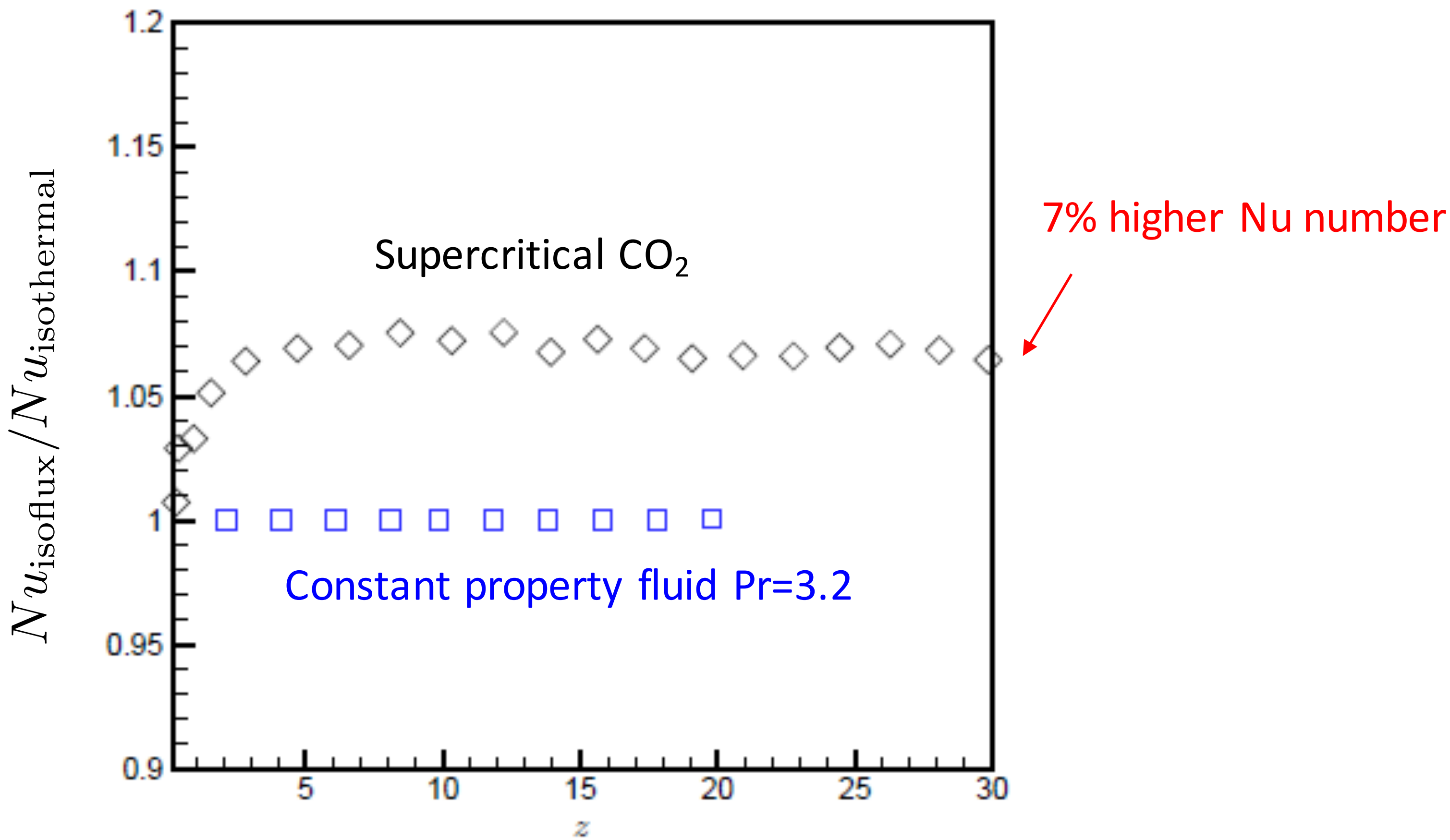
Additional heat flux caused by:

$$\overline{\alpha' \frac{\partial h'}{\partial r}}$$

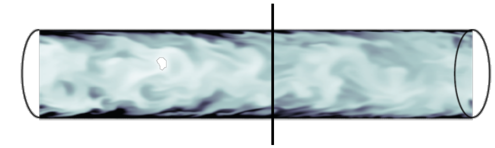


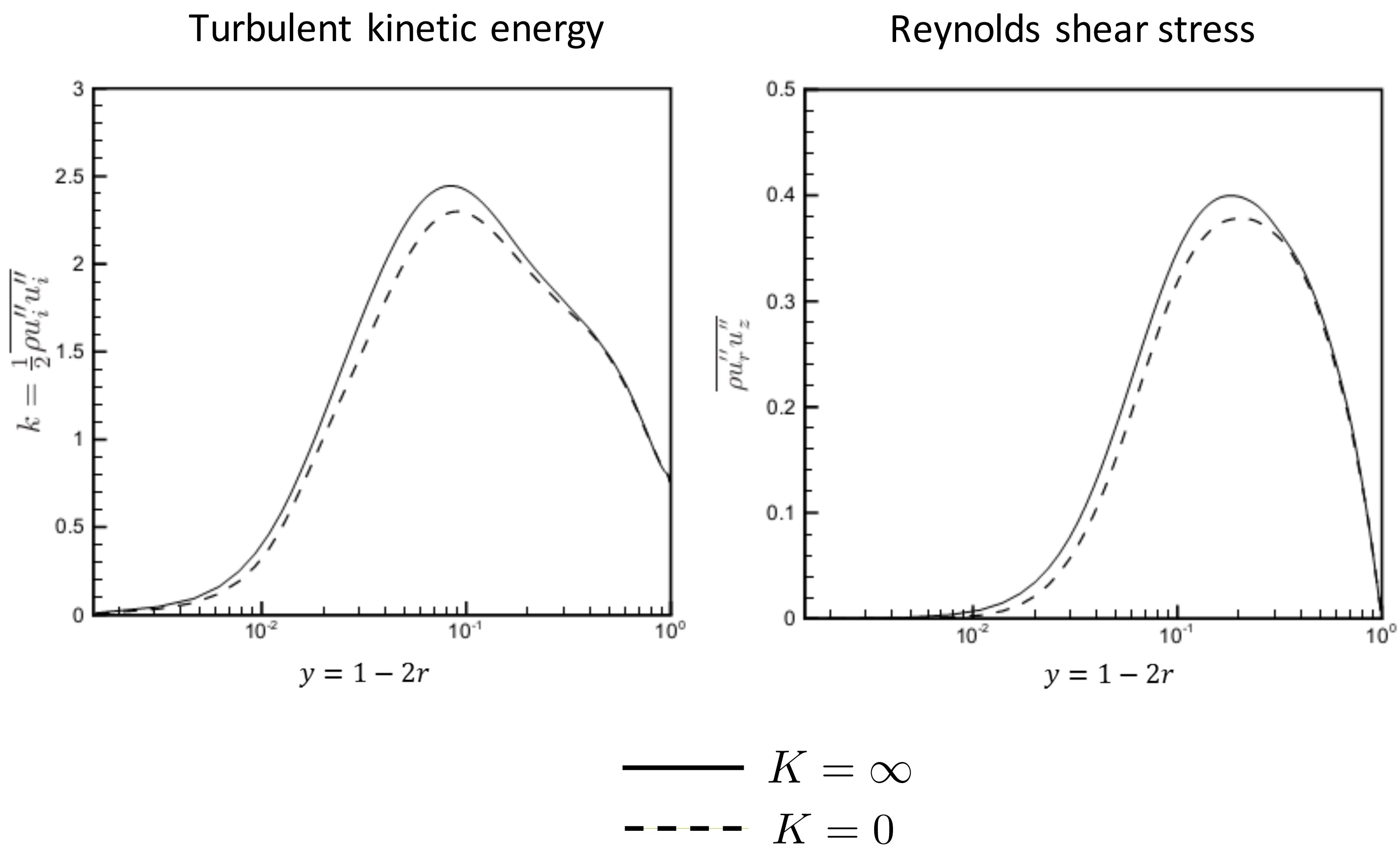
Nusselt number ratio

Nusselt number: $Nu = \frac{\overline{\alpha \frac{\partial h}{\partial r}|_w}}{\lambda_b (T_w - T_b)}$



Turbulent kinetic energy and Reynold shear stress





Decomposed skin friction, FIK identity

(Fukagata, Iwamoto, Kasagi; PoF 2002)

$$C_{f,FIK} = -\frac{2}{\rho_b U_b^2 Re_{\tau 0}}\int_0^R r\overline{\mu}\overline{S}_{rz} r dr + \frac{2}{\rho_b U_b^2}\int_0^R r\overline{\rho u_r'' u_z''} r dr + \left[\frac{1}{\rho_b U_b^2}\int_0^R (R^2-r^2)\frac{\partial\widetilde{\overline{p}}}{\partial z} r dr + \frac{1}{\rho_b U_b^2}\int_0^R (R^2-r^2)\frac{1}{r}\frac{\partial r\widetilde{\overline{\rho}\tilde{u}_r\tilde{u}_z}}{\partial r} r dr \right.$$

$$+ \frac{1}{\rho_b U_b^2}\int_0^R (R^2-r^2)\frac{\partial\widetilde{\overline{\rho}\tilde{u}_z\tilde{u}_z}}{\partial z} r dr + \frac{1}{\rho_b U_b^2}\int_0^R (R^2-r^2)\frac{\partial\widetilde{\overline{\rho u_z'' u_z''}}}{\partial z} r dr - \frac{1}{\rho_b U_b^2 Re_{\tau 0}}\int_0^R (R^2-r^2)\frac{1}{r}\frac{\partial r\widetilde{\overline{\mu' S'}_{rz}}}{\partial r} r dr$$

$$\left. - \frac{1}{\rho_b U_b^2 Re_{\tau 0}}\int_0^R (R^2-r^2)\frac{\partial\widetilde{\overline{\mu}\overline{S}_{zz}}}{\partial z} r dr - \frac{1}{\rho_b U_b^2 Re_{\tau 0}}\int_0^R (R^2-r^2)\frac{\partial\widetilde{\overline{\mu' S'}_{zz}}}{\partial z} r dr \right]$$

Where $$\widetilde{\overline{\Phi}}(r,z) = \overline{\Phi}(r,z) - 8\int_0^R \overline{\Phi}(r,z) r dr$$

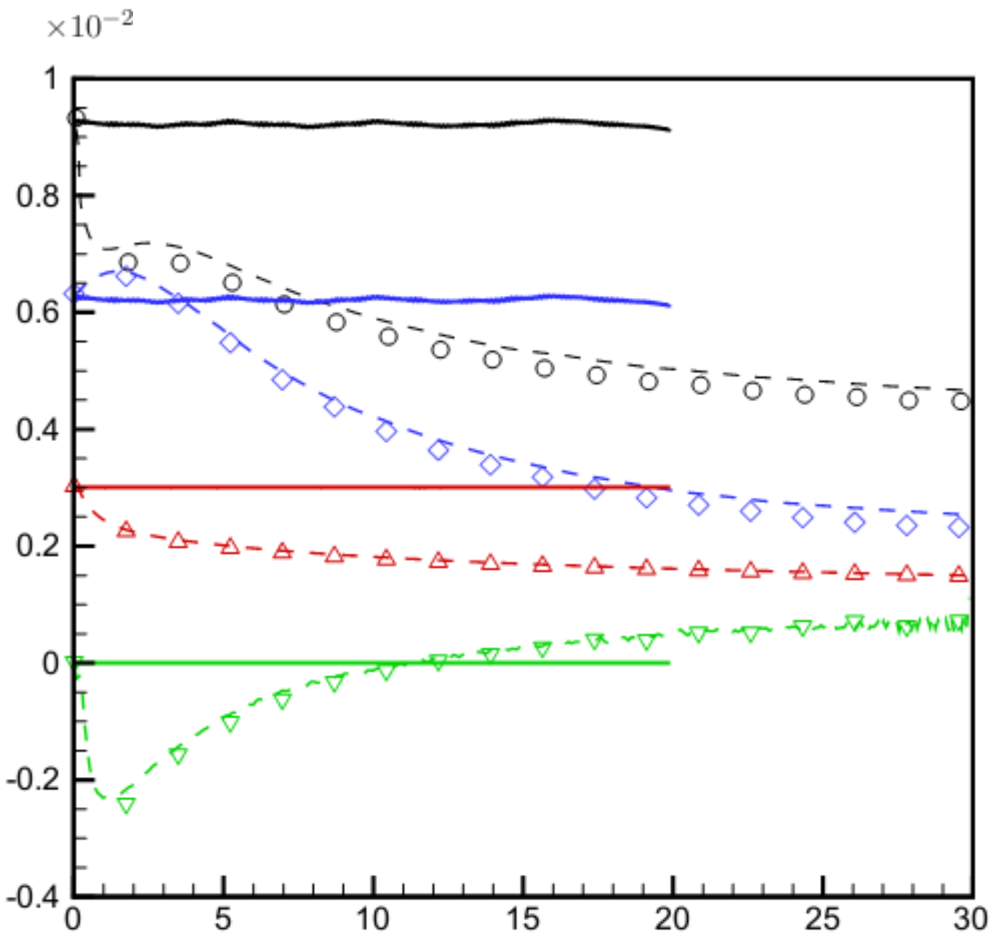
Fully developed pipe flow with constant property fluid

Laminar contribution ⟹ $\frac{16}{Re_b}$

Turbulent contribution

Inhomogeneous contribution

Decomposed skin friction, FIK identity



Total skin friction

Laminar contribution

Turbulent contribution

Inhomogeneous contribution

Dashed lines: Iso-flux

Symbols: Iso-thermal

Decomposed Nusselt number, FIK identity

$$Nu_{FIK} = \frac{32}{\lambda_b(T_w - T_b)} \int_0^R r\overline{\alpha}\frac{\partial \overline{h}}{\partial r} rdr - \frac{32 Re_{\tau 0} Pr_0}{\lambda_b(T_w - T_b)} \int_0^R \overline{r\rho h'' u_r''} rdr - \frac{16 Re_{\tau 0} Pr_0}{\lambda_b(T_w - T_b)} \int_0^R (R^2 - r^2)\frac{1}{r}\frac{\partial r\overline{\rho}\widetilde{h}\widetilde{u}_r}{\partial r} rdr$$

$$- \frac{16 Re_{\tau 0} Pr_0}{\lambda_b(T_w - T_b)} \int_0^R (R^2 - r^2)\frac{\partial \overline{\rho}\widetilde{h}\widetilde{u}_z}{\partial z} rdr - \frac{16 Re_{\tau 0} Pr_0}{\lambda_b(T_w - T_b)} \int_0^R (R^2 - r^2)\frac{\partial \widetilde{\overline{\rho} h'' u_z''}}{\partial z} rdr + \frac{16}{\lambda_b(T_w - T_b)} \int_0^R (R^2 - r^2)\frac{1}{r}\frac{\partial r}{\partial r}\widetilde{\alpha'\overline{\frac{\partial h'}{\partial r}}} rdr$$

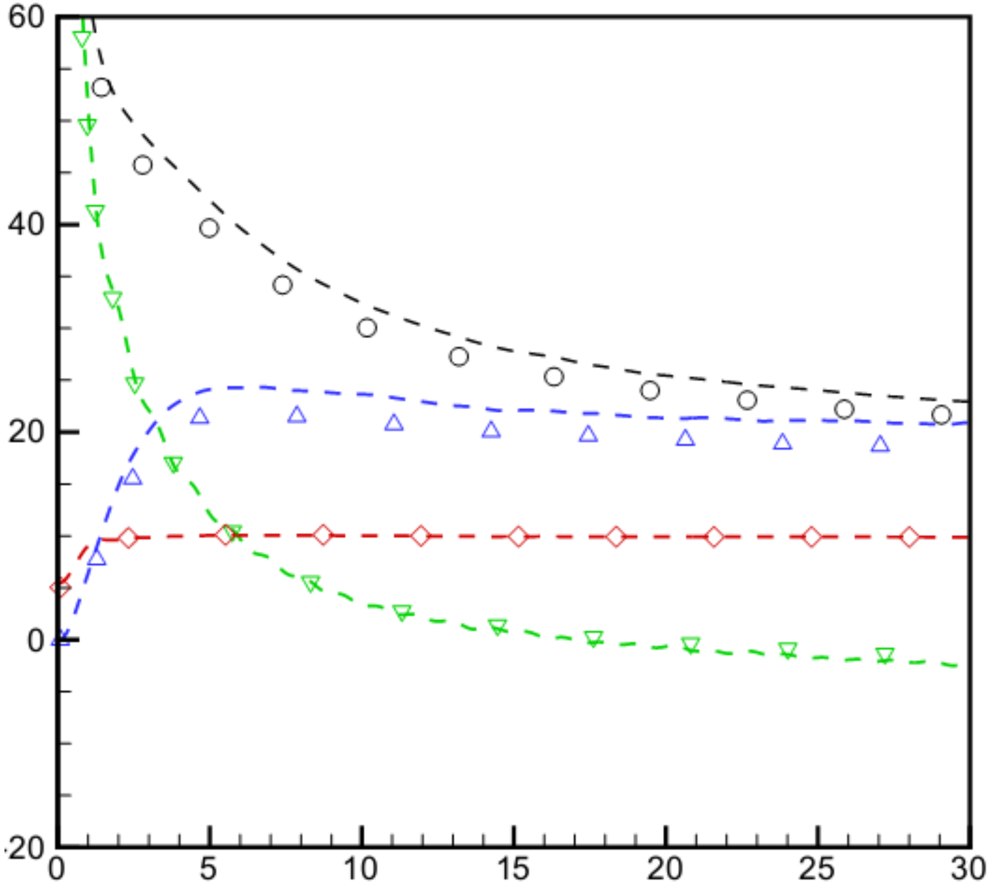
$$+ \frac{16}{\lambda_b(T_w - T_b)} \int_0^R (R^2 - r^2)\frac{\partial}{\partial z}\widetilde{(\overline{\alpha}\frac{\partial \overline{h}}{\partial z})} rdr + \frac{16}{\lambda_b(T_w - T_b)} \int_0^R (R^2 - r^2)\frac{\partial}{\partial z}\widetilde{(\alpha'\overline{\frac{\partial h'}{\partial z}})} rdr$$

Laminar contribution

Turbulent contribution

Inhomogeneous contribution

Decomposed Nusselt number, FIK identity



Total Nusselt number

Laminar contribution

Turbulent contribution

Inhomogeneous contribution

Dashed lines: Iso-flux

Symbols: Iso-thermal

Conclusions

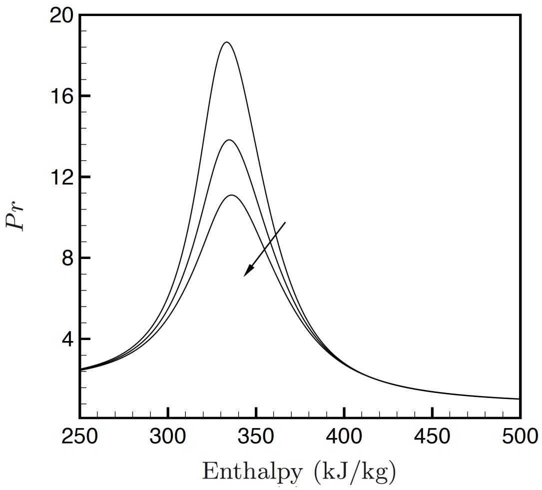
- Thermal effusivity ration has an effect on heat transfer even for Pr > 1 in supercritical flows
- Nusselt number 7% higher for $K = \infty$
- The turbulent heat flux and Reynolds shear stress decrease
- Higher enthalpy fluctuations for $K = \infty$ induce higher density fluctuations, which result in larger velocity fluctuations and thus higher mixing

Thermal activity ratio and Prandtl number examples

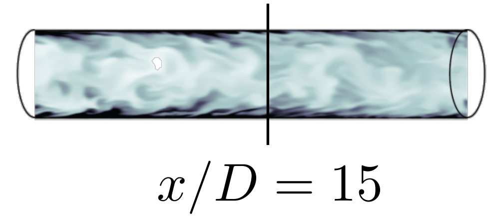
Prandtl number	Air * 0.708	Water * 6.78	$scCO_2$ ~ 4 - 16
Aluminum	0.00025	0.071	
Nickel based alloy	0.00073	0.207	~0.3
Copper	0.00015	0.044	
Glass	0.00419	1.190	
Plexiglas	0.00942	2.680	

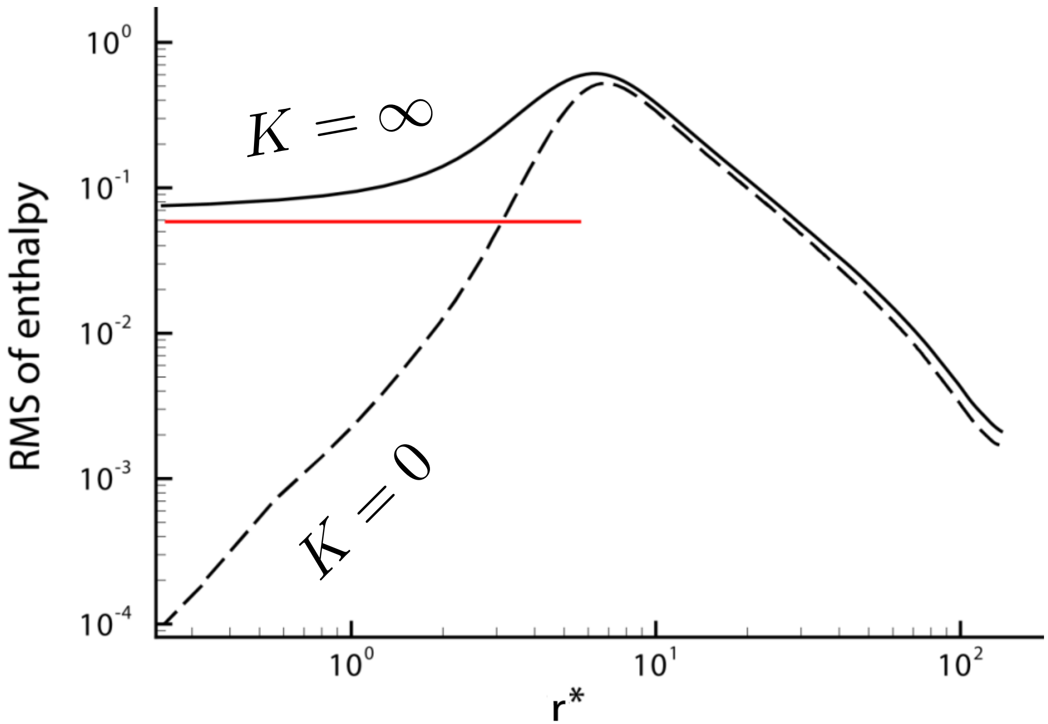
* based on Kasagi et al., Journal of heat transfer, 1989

Prandtl number for CO_2 at 80 bar



Temperature rms values for constant properties

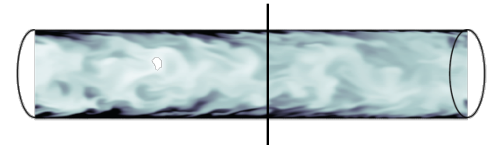




Reynolds decomposition of wall heat flux:

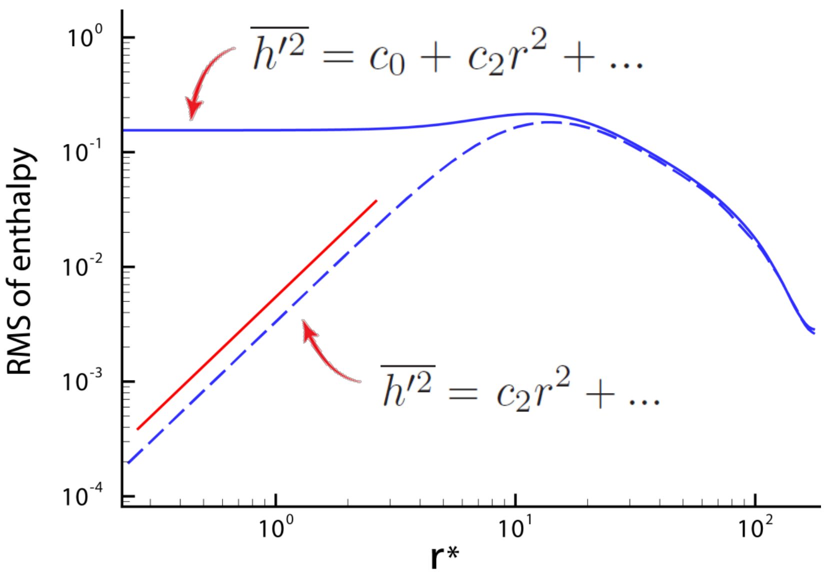
$$Q_w = \overline{\alpha}\frac{\partial \overline{h}}{\partial r}|_w + \alpha'\frac{\partial h}{\partial r}|_w + \overline{\alpha}\frac{\partial h'}{\partial r}|_w \rightarrow \frac{\partial \overline{h'^2}}{\partial r} = -\frac{2}{\overline{\alpha}}\overline{h'\alpha'\frac{\partial h}{\partial r}}$$

Temperature rms values for constant properties



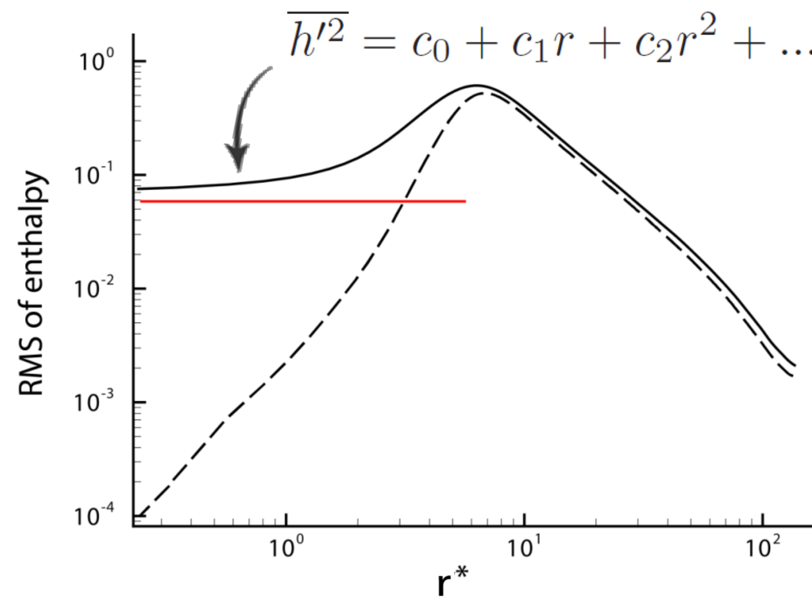
Constant property flow (Pr=3.2)

$$\frac{\partial \overline{h'^2}}{\partial r} = 0$$

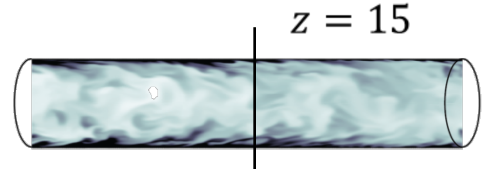


Supercritical fluid flow (Pr_0=3.2)

$$\frac{\partial \overline{h'^2}}{\partial r} = -\frac{2}{\overline{\alpha}} \overline{h'\alpha' \frac{\partial h}{\partial r}}$$



Radial heat fluxes

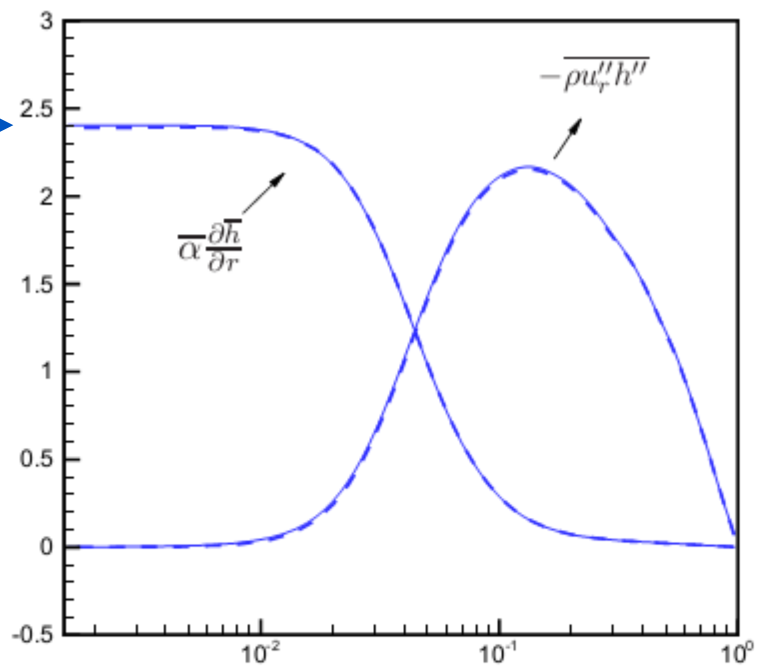


Total radial heat flux:

$$q_{r,tot} = \overline{\alpha}\frac{\partial \overline{h}}{\partial r} + \overline{\alpha'\frac{\partial h'}{\partial r}} - \overline{\rho u_r'' h''}$$

Constant property (CP) flow

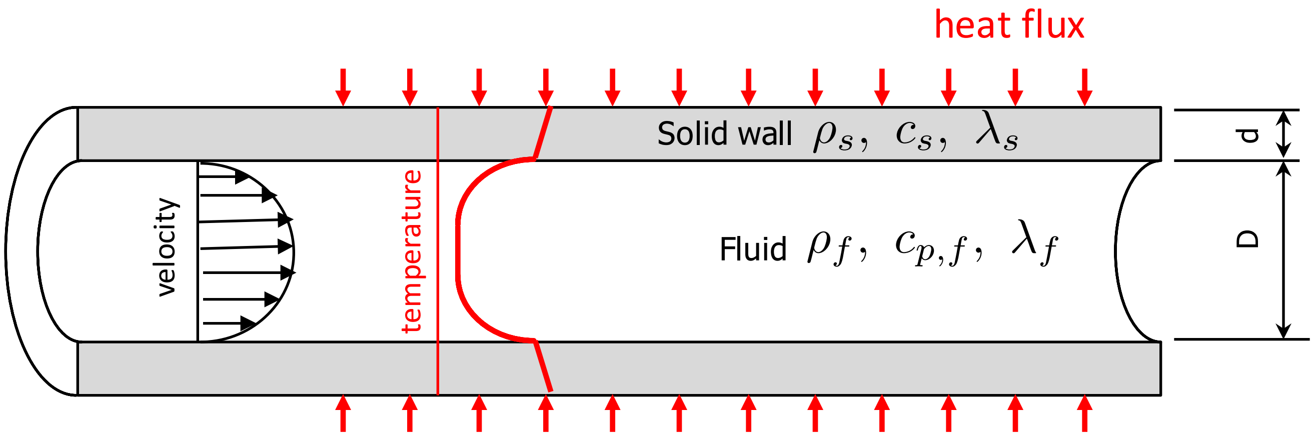
Specified heat flux = 2.4



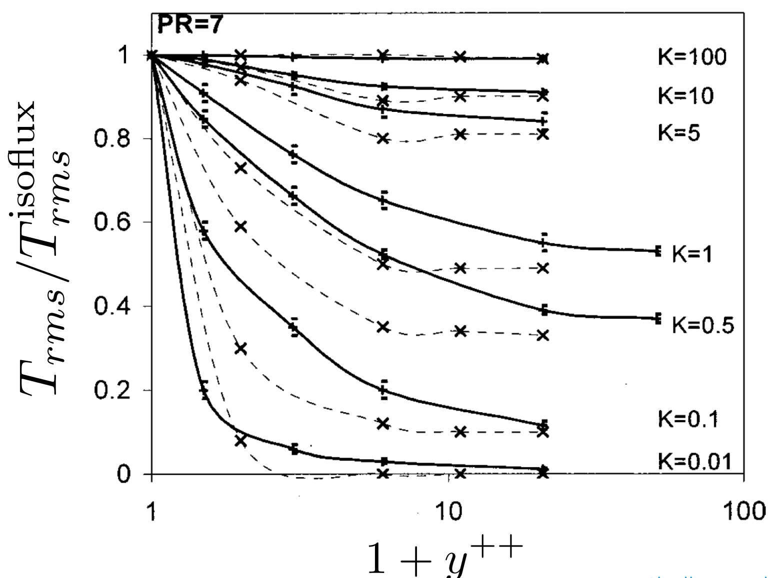
Iso-flux

Iso-thermal

Effect of wall thickness on temperature fluctuations



From Tiselj et al. 2001, JHT



Dimensionless wall thickness:

$$y^{++} = \sqrt{\lambda_f/\lambda_s} \; y^+$$

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