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Two-Layer Model for the Heat Transfer to Supercritical CO_2

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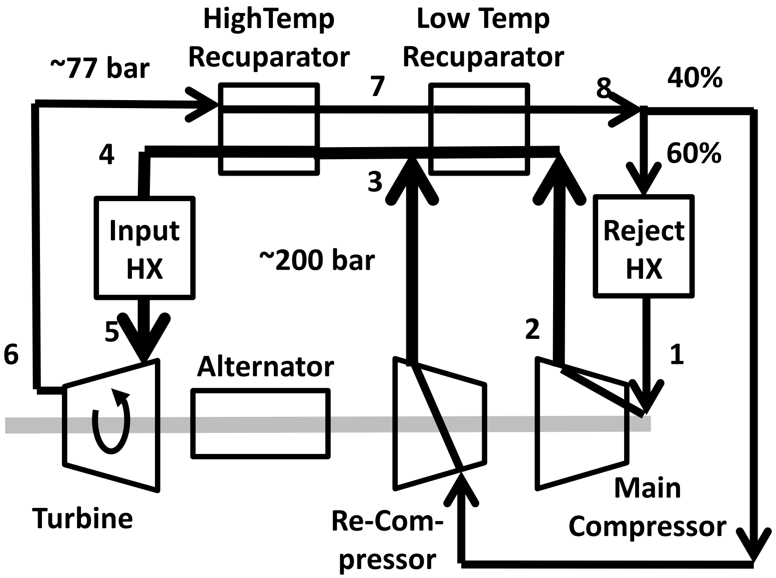
Outline

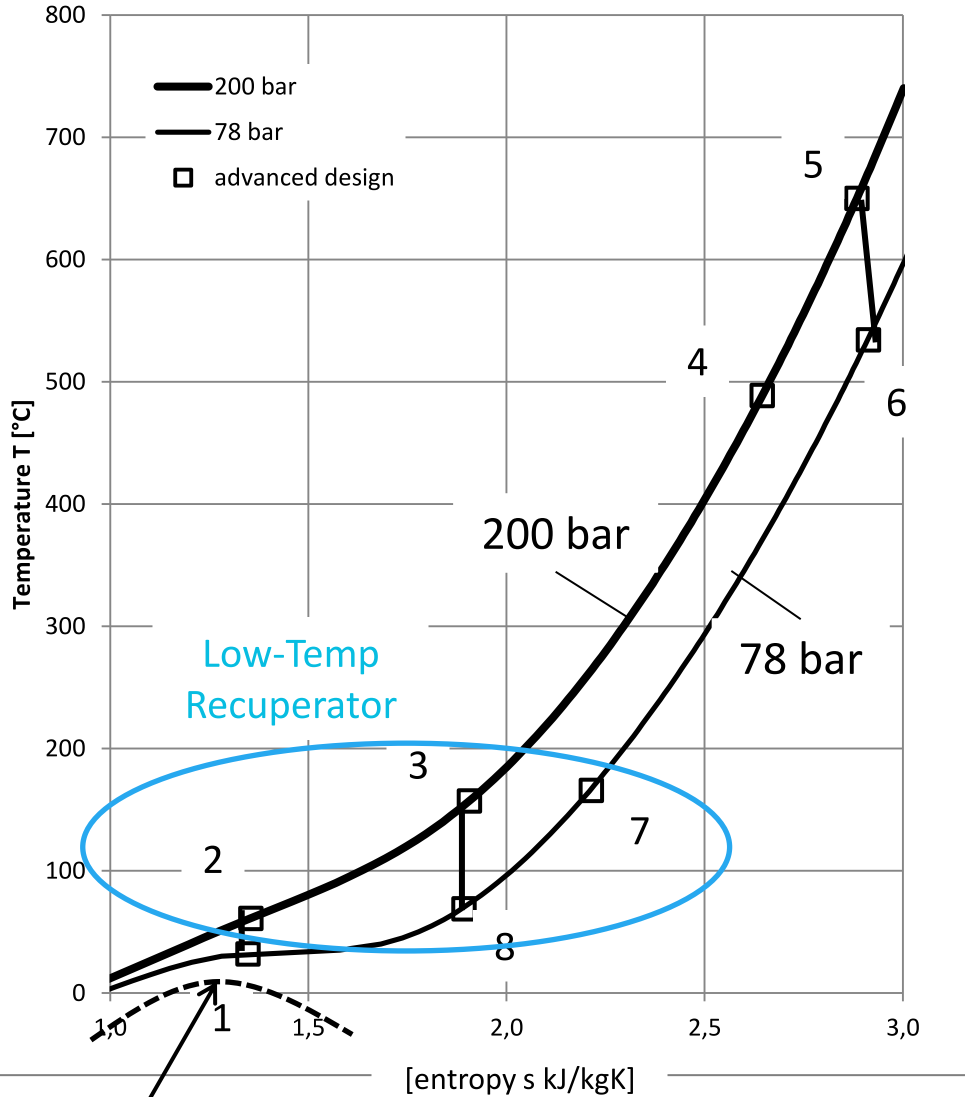
- Introduction
- Two-Layer Model for a Supercritical Fluid
- Extension to Take Account of Buoyancy
- Application to the Low-Temperature Recuperator
- Application to the Reject Heat Exchanger
- Conclusion

Supercritical CO_2-Loop for Energy Conversion

MIT-Study



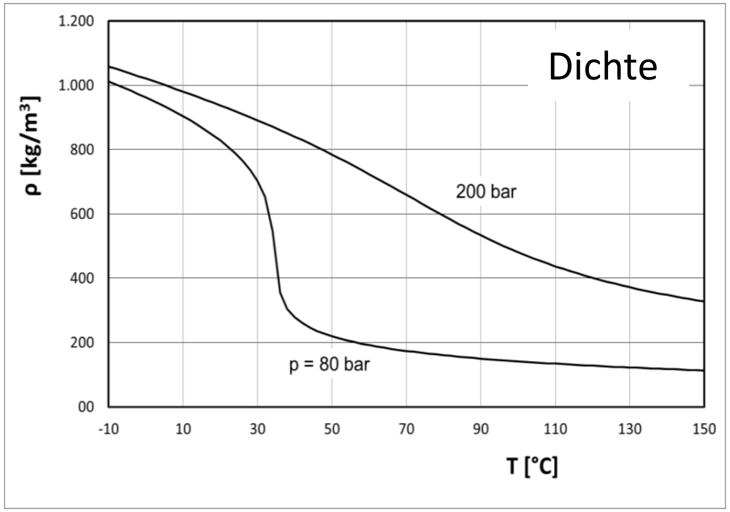
V. Dostal, M.J. Driscoll, P. Hejzlar: A Supercritical Carbon Dioxide Cycle for Next Generation Nuclear Reactors , MIT-ANP-TR-100 (2004)

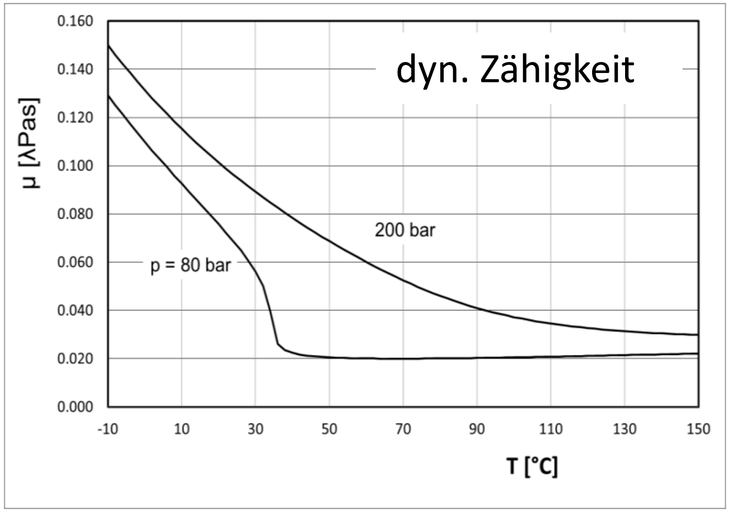


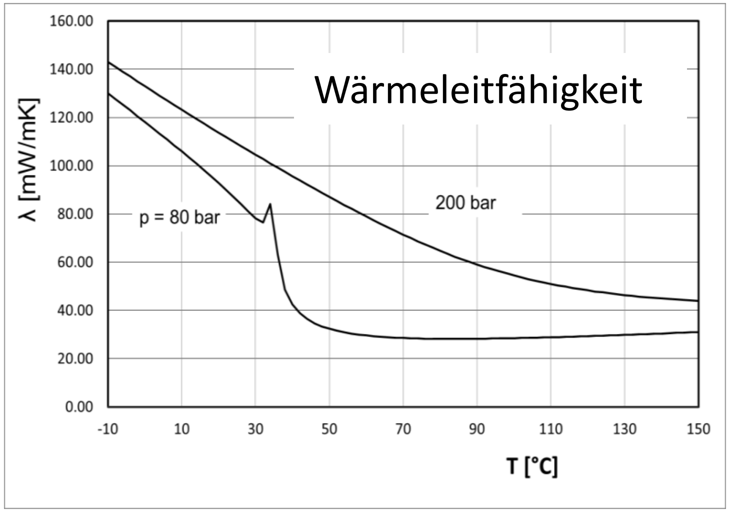
critical point

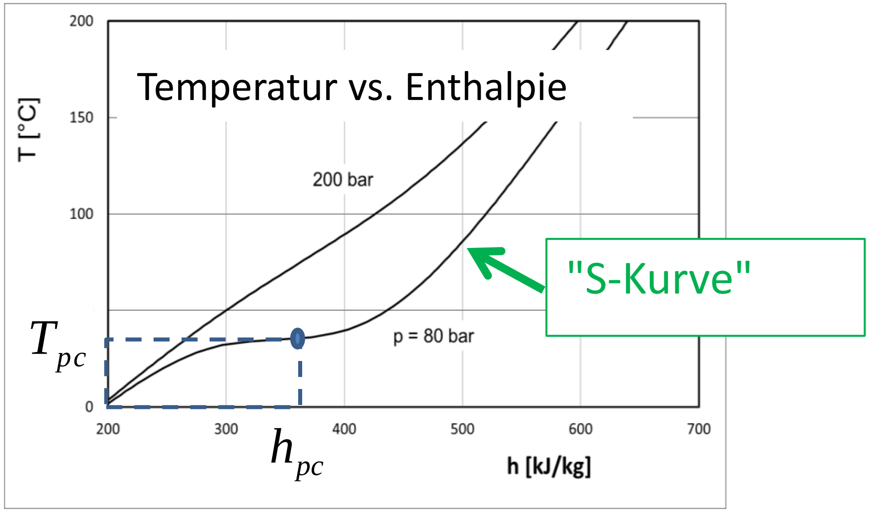
Fluid Properties

oc CO2









W. Lemmon, Marcia L. Huber, and Mark O. McLinden
Thermophysical Properties Division
National Institute of Standards and Technology
Program. REFPROP (2014)

vom Dittus-Boelter Typ

$$Nu_t = C_1 Re_t^{m1} Pr_t^{m2} \left(\frac{\rho_w}{\rho_b}\right)^{m3} \left(\frac{\overline{c}_p}{c_{pt}}\right)^{m4} C_2$$

eferenz	t	C_1	m_1	m_2	m_3	m_4	C_2
ishop et al. [27]	*b*	0,0069	0,9	0,66	0,43	0,66	(1+2,4*d*/*l*)
ittus-Boelter, nach [87]	*b*	0,023	0,8	0,4			
riem [41]		0,0169	0,8356				$\widetilde{Pr}^{0,432} \cdot \Phi$
ackson und Hall [58]	*b*	0,0183	0,82	0,5	0,3	*n*	
wenson et al. [26]	*w*	0,00459	0,923	0,613	0,231	0,613	
atts und Chou [38]	*b*	0,021	0,8	0,55	0,35	0,55	ϕ
amagata et al. [36]	*b*	0,0135	0,85	0,8			F_c

diese Korrelationen
sind oft sehr ungenau

andere Typen

Petukhov et al. [21]

$$Nu_b = \frac{(\xi_0/8) Re_b \overline{Pr}_b}{\left[12,7\sqrt{\xi_0/8}\left(\overline{Pr}_b^{2/3}-1\right)+1,07\right]} \left(\frac{\eta_b}{\eta_w}\right)^{0,11} \left(\frac{\lambda_b}{\lambda_w}\right)^{-0,33} \Bigg(\frac{\overline{c}_p}{c_{pb}}$$

Razumovskiy et al. [39]

$$Nu_b = \frac{\left(\xi_{fr}/8\right) Re_b \overline{Pr}_b}{1,07+12,7\sqrt{\xi_{fr}/8}\left(\overline{Pr}_b^{2/3}-1\right)} \left(\frac{\overline{c}_p}{c_{pb}}\right)^{0,65}$$

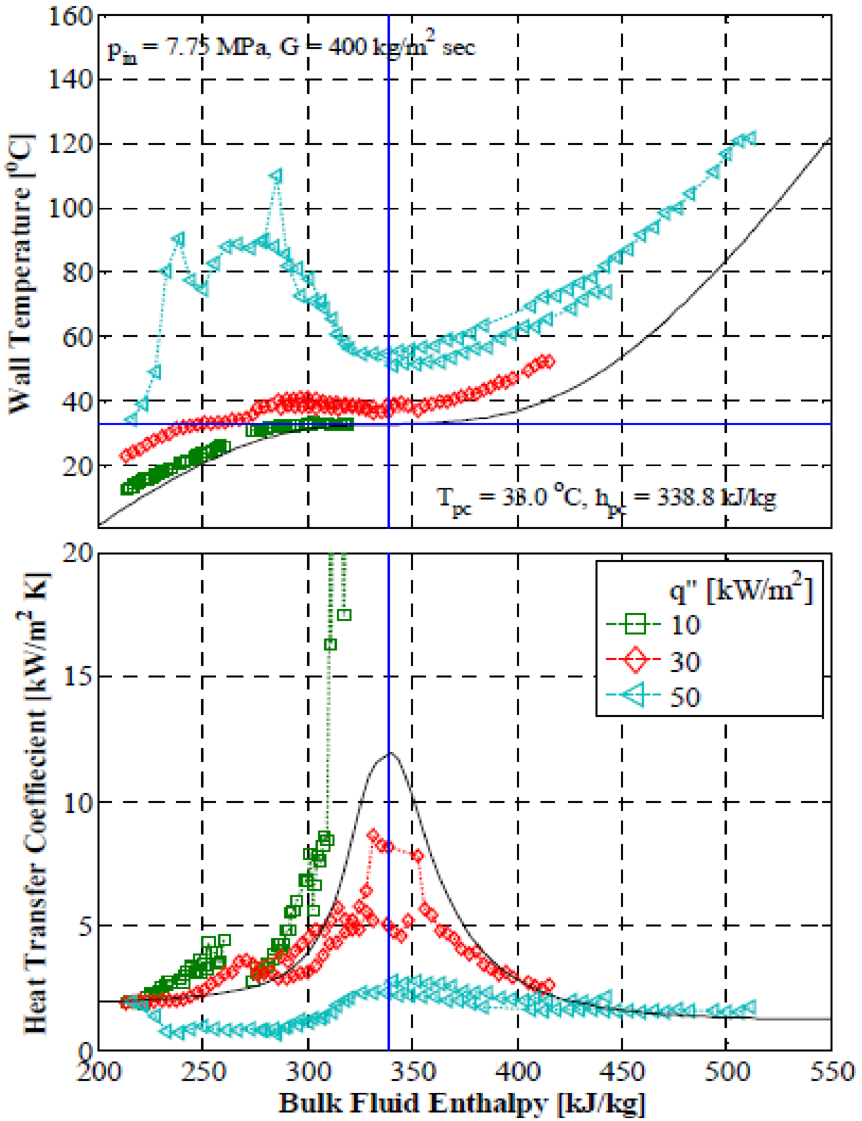
Grass et al. [57]

$$Nu_b = \frac{\left(\xi_0/8\right) Re_b Pr_b}{1,07+12,7\sqrt{(\xi_0/8)}\left(Pr_G^{2/3}\frac{c_{pb}}{c_{pG}}-1\right)}$$

$$\xi_{fr} = \xi_0 \left(\frac{\eta_w}{\eta_b}\frac{\rho_w}{\rho_b}\right)^{0,18} \quad ; \quad \xi_0 = \left(1,82\log\left(\frac{Re_b}{8}\right)\right)^{-2}$$

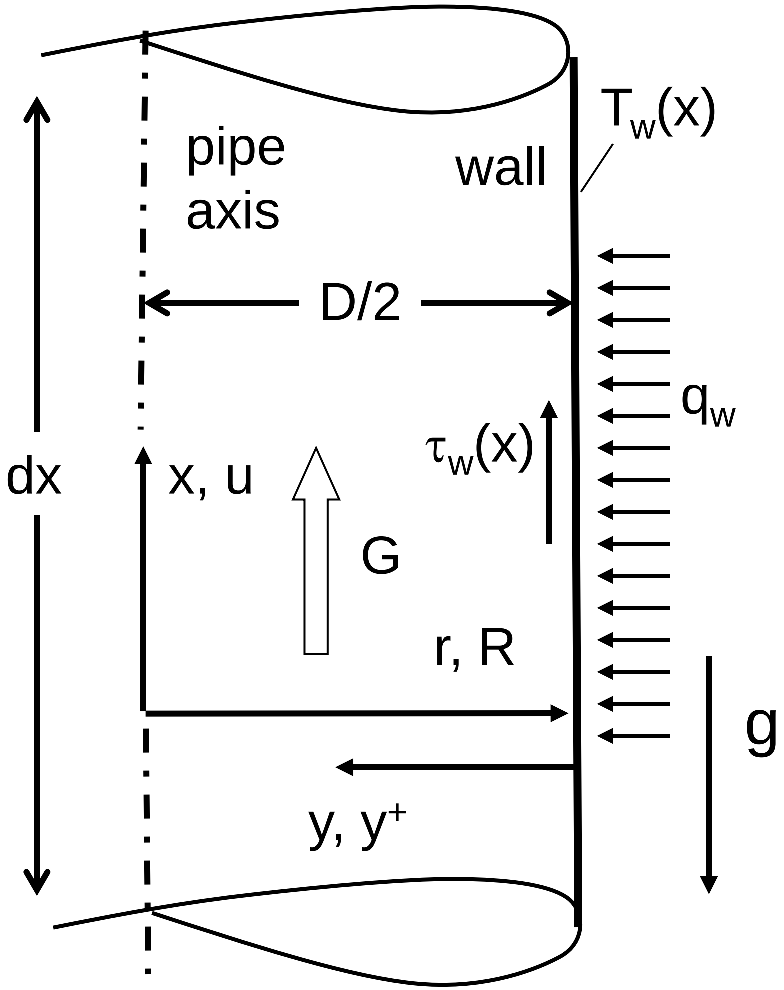
Experiments by Kim et al.

77.5 bar heated wall



Integration Domain an Governing Equations

Sub-Headline



Theoretical Description Methods:

- Lookup Table
- CFD (RANS and DNS)
- 1-dimensional theory by Laurien 2012
- Explicit or implicit formulas (model equations), correlations

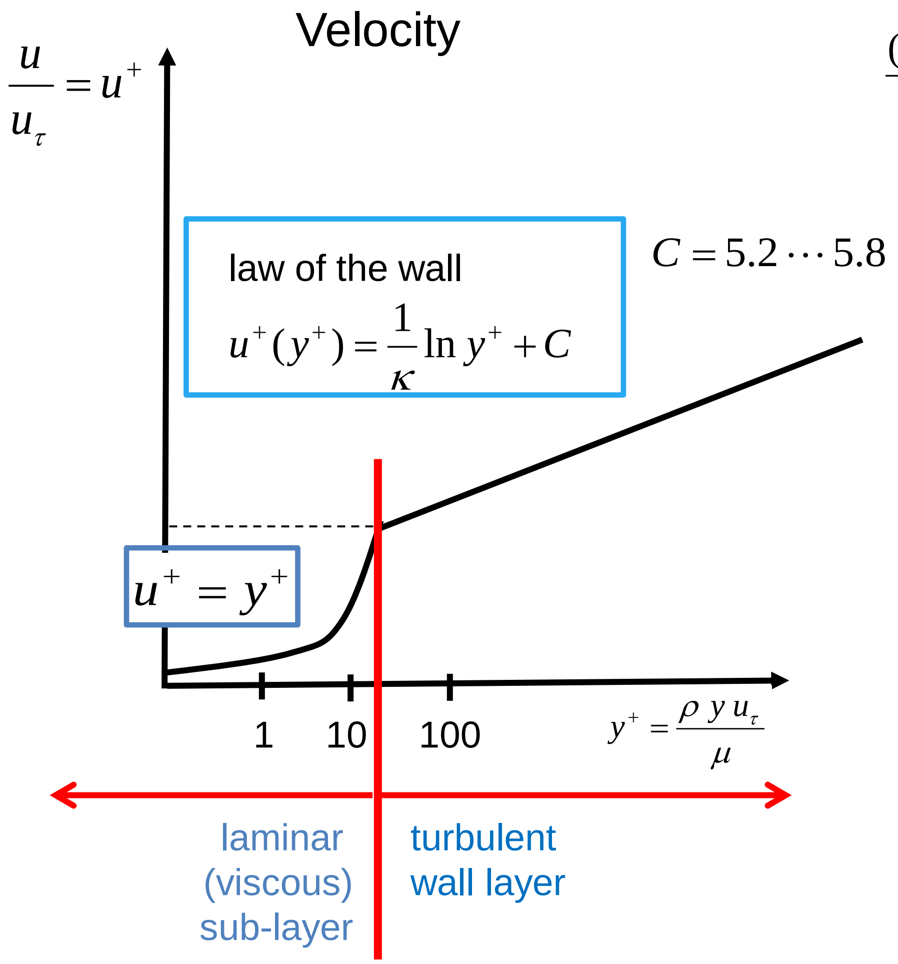
$$Nu = a \cdot \mathrm{Re}^{e1}\,\mathrm{Pr}^{e2}\left(\frac{\lambda_b}{\lambda_w}\right)^{e3}\left(\frac{\mu_b}{\mu_w}\right)^{e4}\left(\frac{\rho_b}{\rho_w}\right)^{e5}\left(\frac{c_{pb}}{c_{wb}}\right)^{e6}$$

Two-Layer Method

Constant Properties

$$\mathrm{Re}_\tau = \frac{\rho D u_\tau}{\mu} = \sqrt{\frac{c_f}{8}}\,\mathrm{Re} \rightarrow u_\tau$$

Velocity



$$\frac{u}{u_\tau} = u^+$$

law of the wall

$$u^+(y^+) = \frac{1}{\kappa}\ln y^+ + C$$

$$C = 5.2 \cdots 5.8$$

$$u^+ = y^+$$

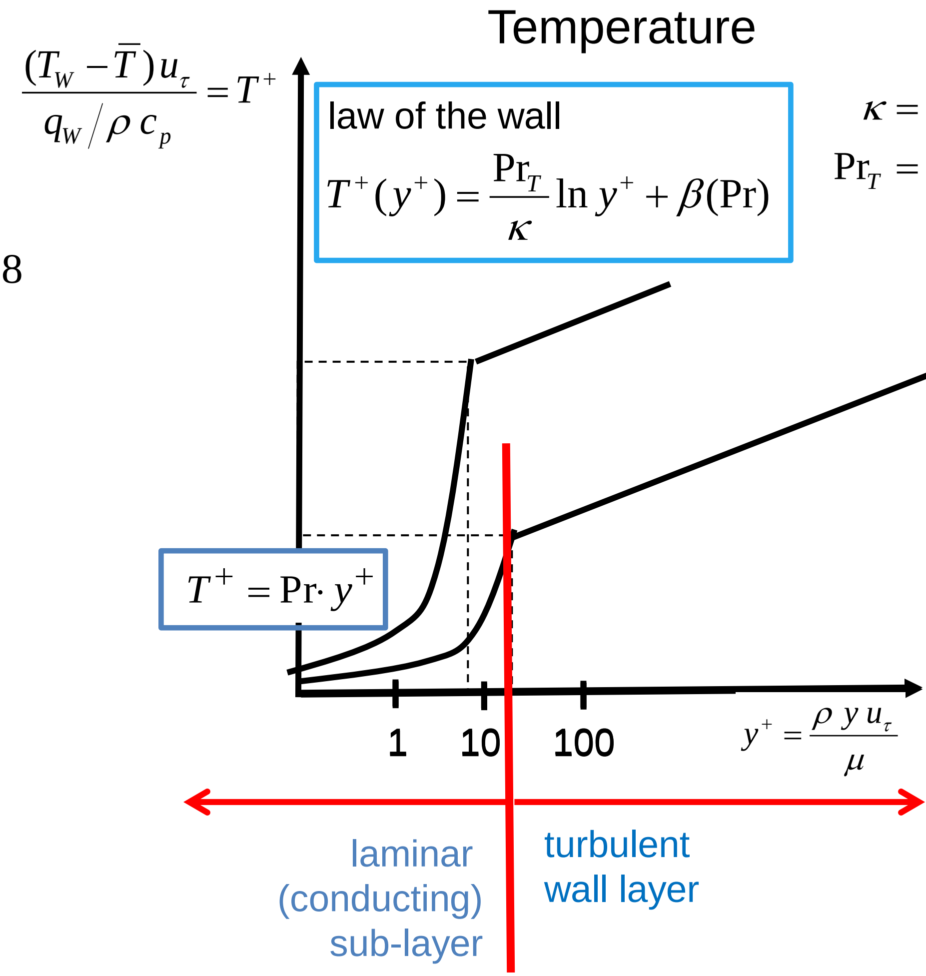
$$y^+ = \frac{\rho\, y\, u_\tau}{\mu}$$

laminar
(viscous)
sub-layer

turbulent
wall layer

$$y^+_{vs} \approx 11.6$$

Temperature



$$\frac{(T_W - \bar{T})\,u_\tau}{q_W / \rho\, c_p} = T^+$$

law of the wall

$$T^+(y^+) = \frac{\mathrm{Pr}_T}{\kappa}\ln y^+ + \beta(\mathrm{Pr})$$

$$\kappa =$$

$$\mathrm{Pr}_T =$$

$$T^+ = \mathrm{Pr}\cdot y^+$$

$$y^+ = \frac{\rho\, y\, u_\tau}{\mu}$$

laminar
(conducting)
sub-layer

turbulent
wall layer

$$y^+_{cs} = \frac{y^+_{vs}}{\mathrm{Pr}^{1/3}}$$

Headline

Sub-Headline

Scaling with bulk quantities

$$y^{+w} = \frac{\rho_b \, y \, u_{\tau w}}{\mu_b} \qquad u_{\tau b} = \sqrt{\frac{\tau_w}{\rho_b}}$$

Scaling with wall quantities

$$y^{+w} = \frac{\rho_w \, y \, u_{\tau w}}{\mu_w} \qquad u_{\tau w} = \sqrt{\frac{\tau_w}{\rho_w}}$$

Friction

created at the wall

$$\mathrm{Re}_w = \frac{\rho_w \, u_m \, D}{\mu_w} \quad \Big| \quad c'_f = \frac{\rho_b}{\rho_w} c_f$$

$$\frac{1}{\sqrt{c'_f}} = -2 \log_{10}\left(\frac{\varepsilon / D}{3.7} + \frac{2.51}{\mathrm{Re}_w \sqrt{c'_f}} \right)$$

⇨

$$\tau_w = \frac{c_f}{8} \rho_b u_m^2$$

wall shear stress

Heat Transfer

turbulent Layer (*turb*)

$$\Delta T^{+b}_{turb} = \frac{\Delta T_{turb} \, u_{\tau b}}{q_W / \rho_b \, c_b}$$

$$\Delta T^{+b}_{turb} = \frac{\mathrm{Pr}_T}{\kappa} \left(\ln R^{+b} - \ln y^{+b}_{cs} \right)$$

conducting sublayer (*cs*)

$$\Delta T^{+}_{cs} = \frac{\Delta T_{cs} \, u_{\tau w}}{q_W / \rho_w \, c_w}$$

$$\Delta T^{+w}_{cs} = \mathrm{Pr}_w^{2/3} \, y^{+w}_{vs}$$

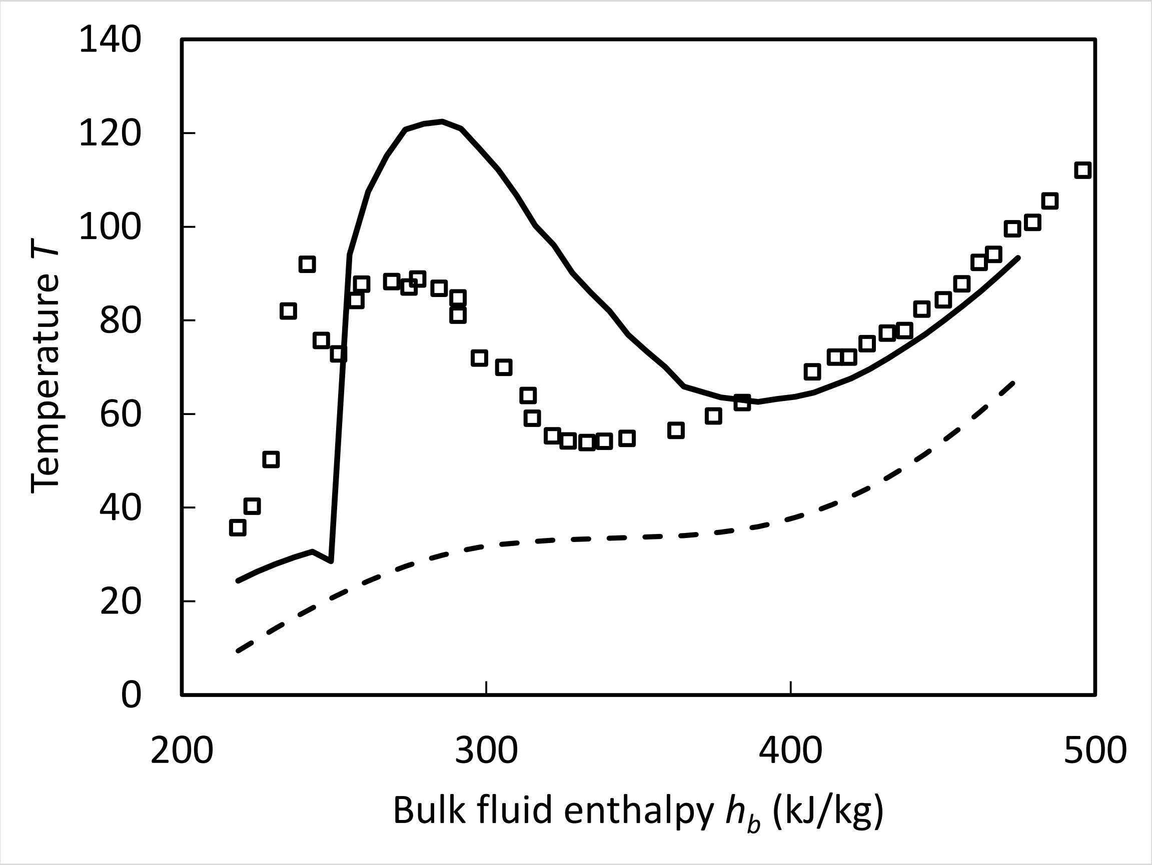
⇨

$$T_{cs} = T_b + \Delta T_{turb}$$
$$T_w = T_{cs} + \Delta T_{cs}$$

temperature increase

Result of the Original Method

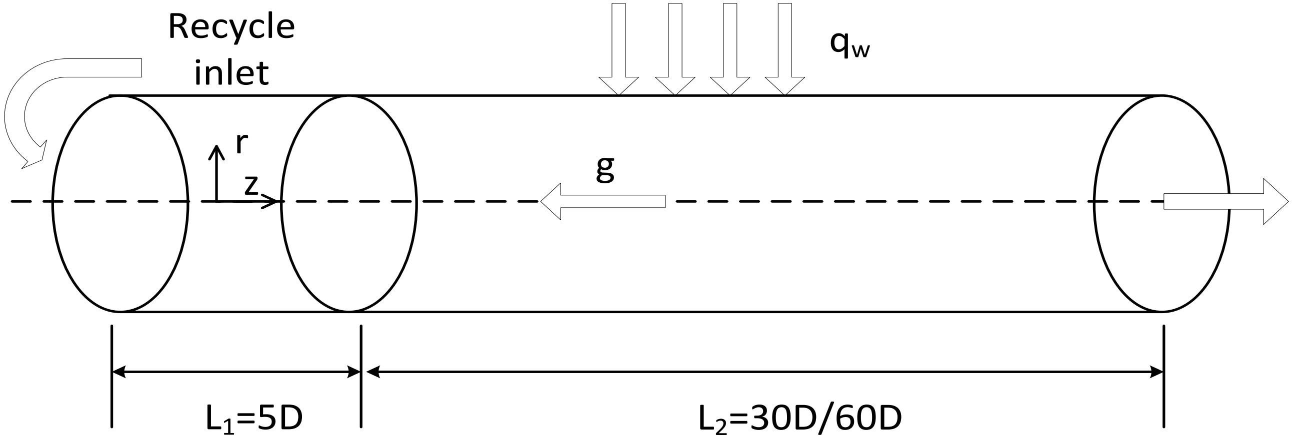
Experiment Li et al.



deterioration can be predicted, but some iaccuracies remain

Headline

Sub-Headline

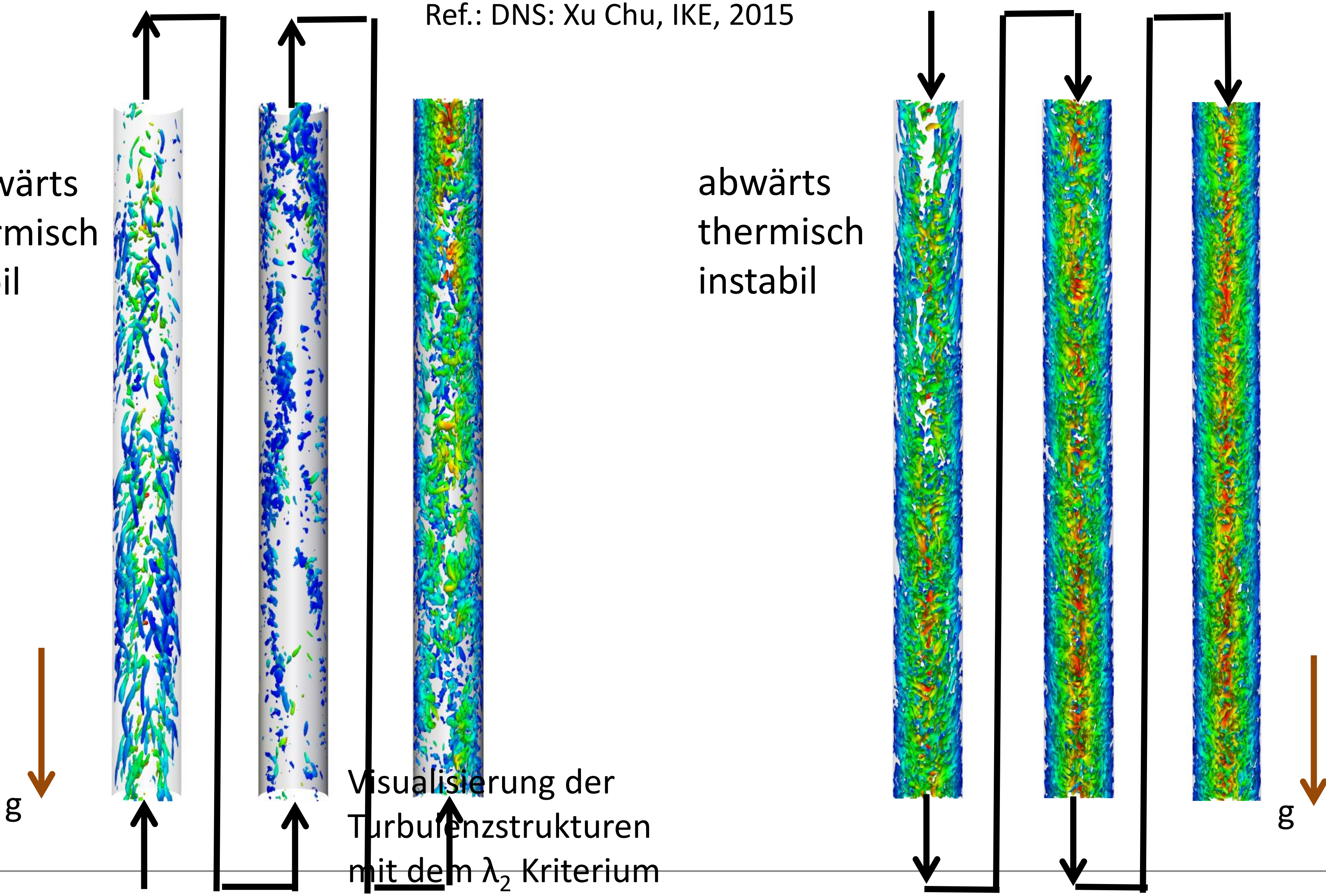


Re	5400	9000
L_2	30D	60D
Δr_1^+	0.16	0.17
Δz^+	7.6	6.6
$(R\Delta\theta)^+$	9.6	9.5
no of cells	22-70 Mio.	150 Mio.

Autor: Xu Chu, IKE
Code: OpenFoam
Cray XC-40 'HazelHen' des HLRS
ca. 1000 CPU-cores

Re = 5400 Vertikales beheiztes Rohr

Ref.: DNS: Xu Chu, IKE, 2015

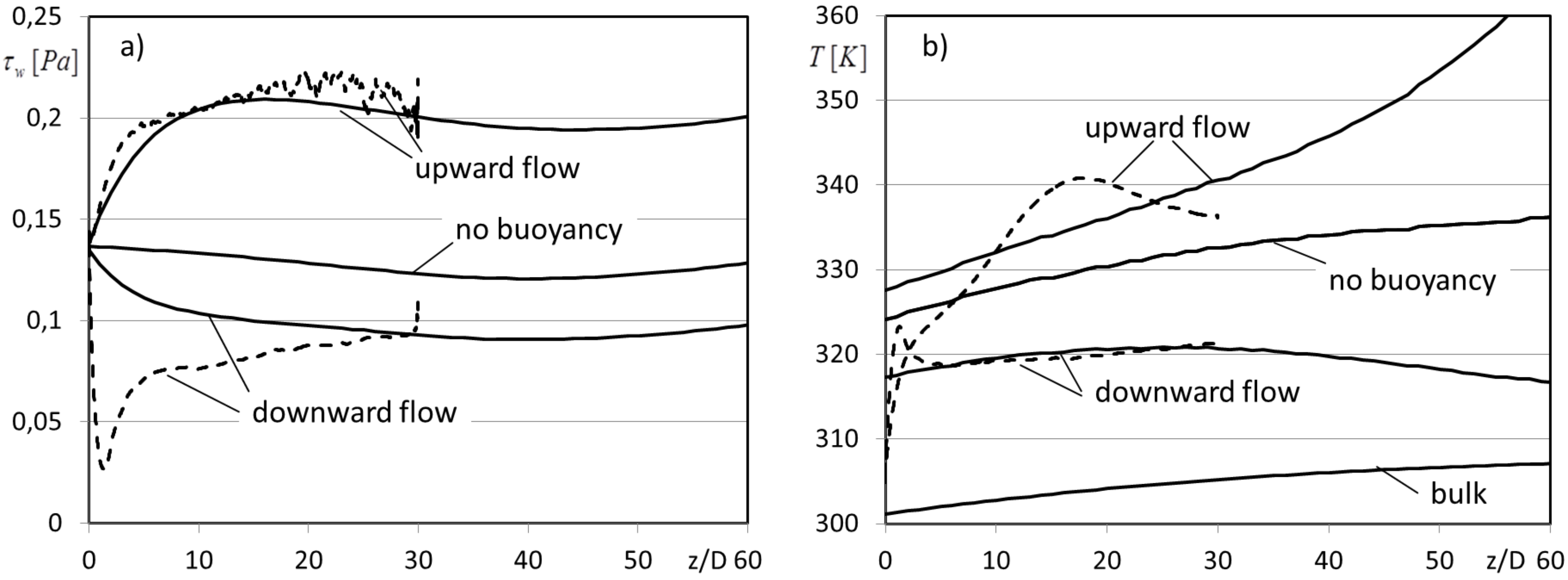


aufwärts
thermisch
stabil

abwärts
thermisch
instabil

Visualisierung der
Turbulenzstrukturen
mit dem λ_2 Kriterium

Calibration of the Method



Extension of the Method

to take account of acceleration and buoyancy

Implementation in the method

Acceleration Parameter

$$K_v = \frac{4 q_w \mu_b u_m}{G D \rho_b^2} \left.\frac{\partial \rho}{\partial h}\right|_b \quad ; \quad \left.\frac{\partial \rho}{\partial h}\right|_b = \frac{\rho_b \beta_b}{c_{pb}}$$

$$y_{vs}^{+w} = 11.8 + c_v K_v$$

Parameter for the 'structural' effect of buoyancy

$$Ri_b = \frac{Gr_b}{\mathrm{Re}_b^2}$$

$$y_{cs}^{+w} = \frac{11.8}{\mathrm{Pr}_{cs}^{1/3}} - c_{buoy,in} \cdot Ri_b$$

Parameter for the 'external' effect of buoyancy

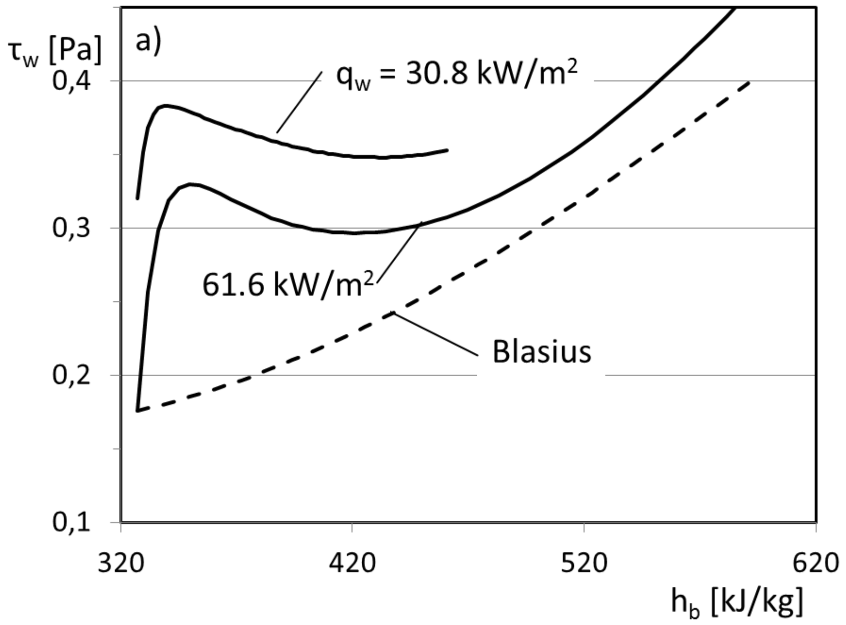
$$Gr_w = \frac{g \beta_w \, \rho_w^2 D^3 \left|T_b - T_w\right|}{\mu_w^2}$$

$$\tau_{w,\mathrm{mod}} = \tau_w + \left(c_{buoy,ex} \times 10^{-7} \; Gr_w\right)\left(1 - e^{-z/5D}\right)$$

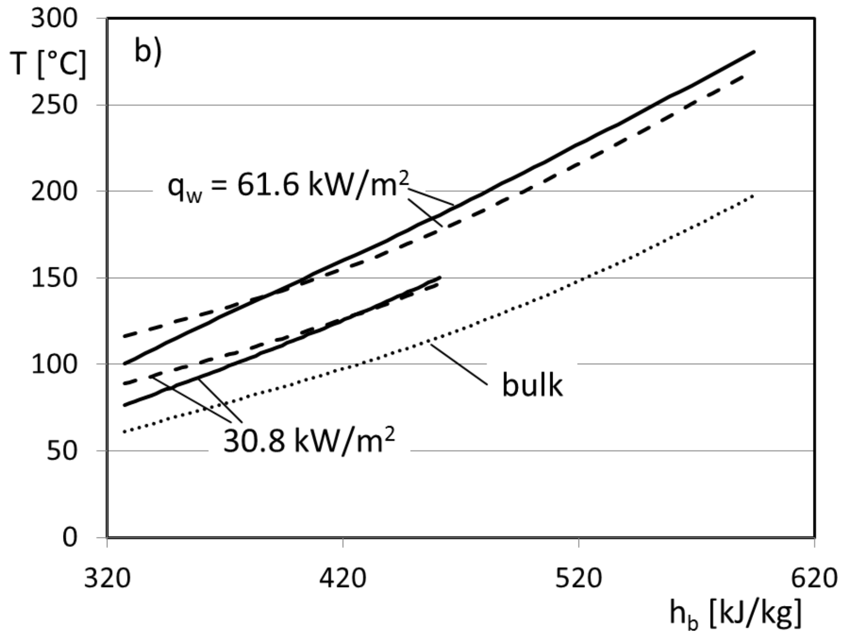
Low-Temperature Recuperator, Secondary Side

200 bar upward flow

wall shear stress



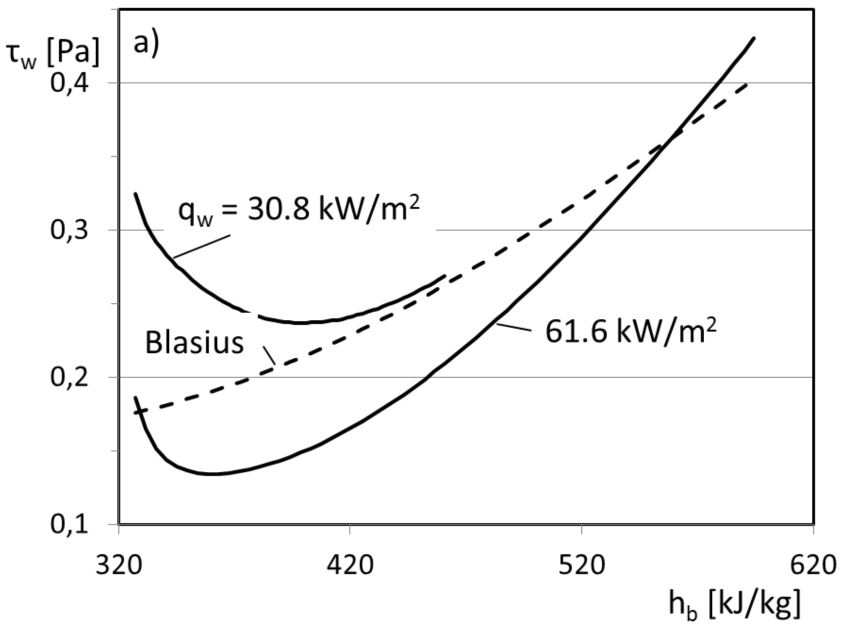
wall temperature

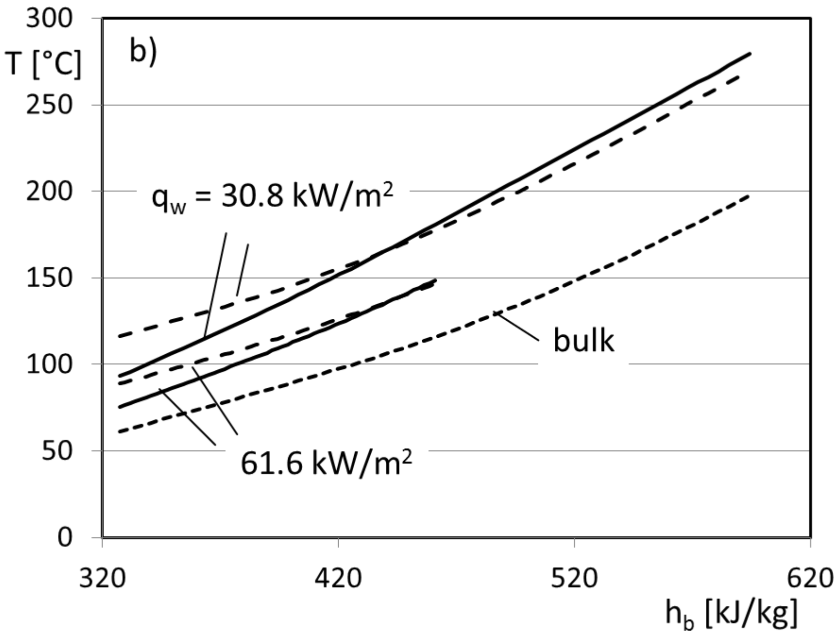


Low-Temperature Recuperator, Secondary Side

200 bar downward flow

wall shear stress **wall temperature**



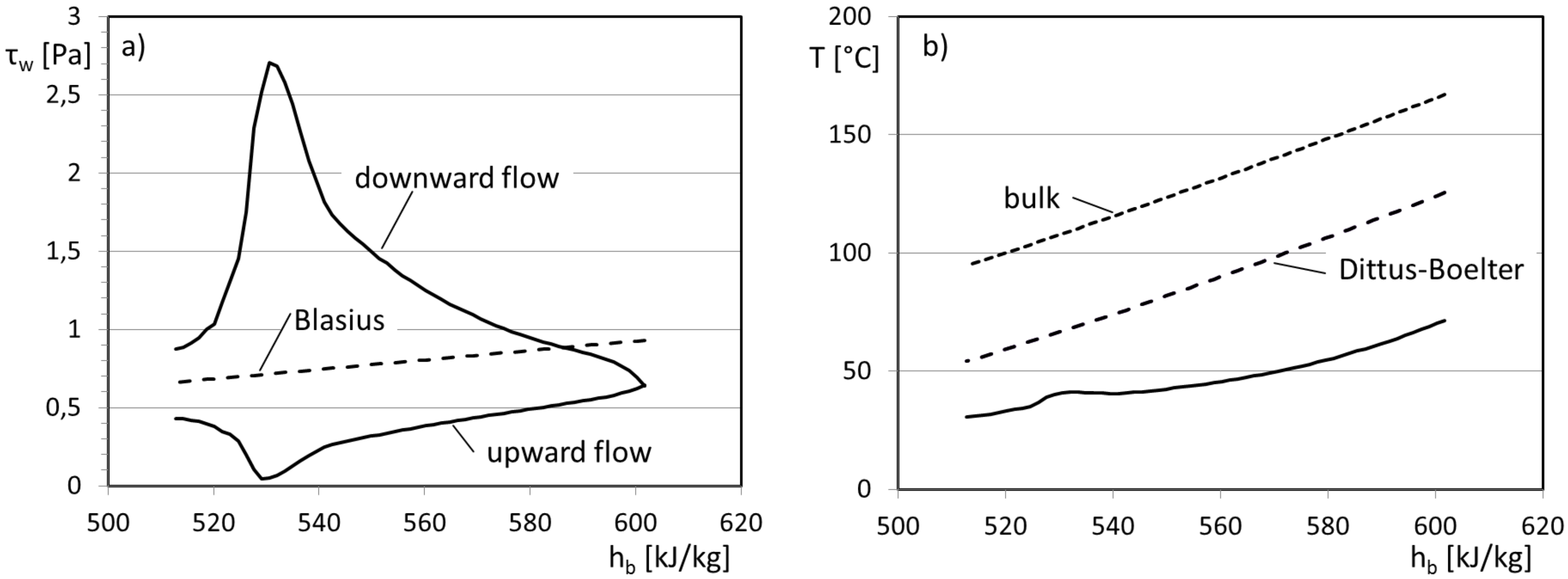


Low-Temperature Recuperator, Primary Side

80 bar cooling

wall shear stress

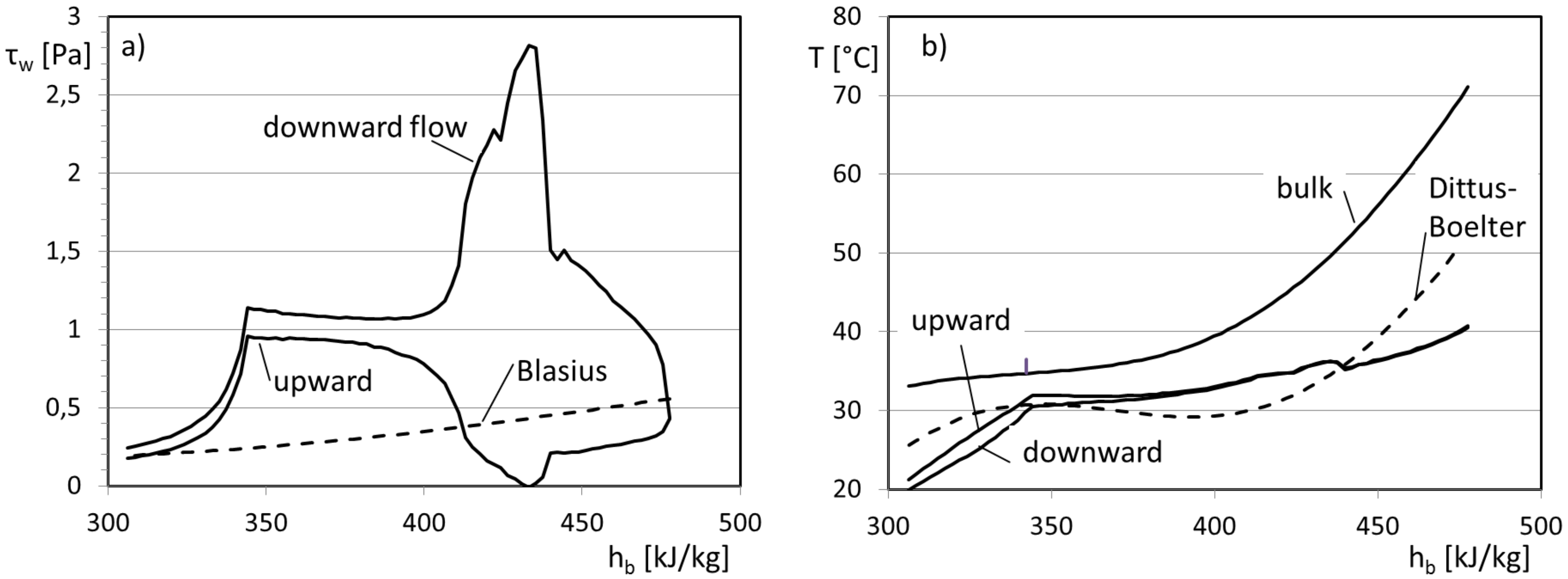
wall temperature



Reject-Heat Exchanger

80 bar cooling

wall shear stress **wall temperature**



Conclusion

- The two-layer model has been modified to take account of buoyancy and acceleration in a vertical (upeard or downward) supercritical pipe flow at low Reynolds numbers (Re < 10000)
- Wall temperature and wall shear stress are determined as a function of pressure, pipe diameter, wall heat flux (heating or cooling), mass flux, and bulk enthalpy
- Empirical model parameters have been determined using two results of Direct Numerical Simulations
  - must be extended by further work (in progress)
- Model is applied to flows in the low-temperature recuperator and the reject-heat exchanger